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# The dynamics of **elongated** earthquake ruptures

Huihui Weng and Jean-Paul Ampuero

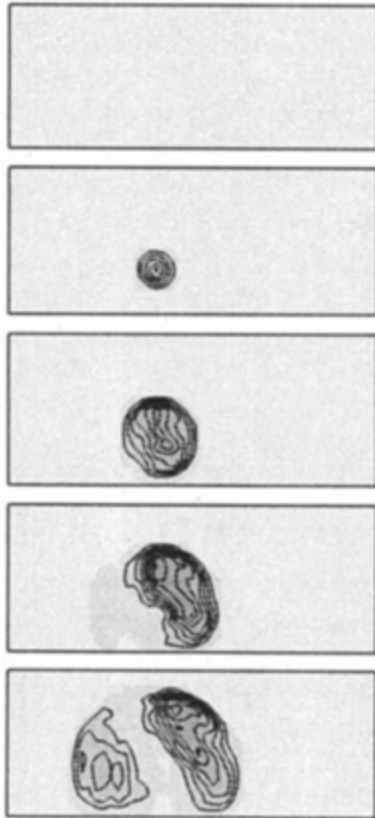
Université Côte d'Azur, IRD, Géoazur

Smolenice Castle, Slovakia

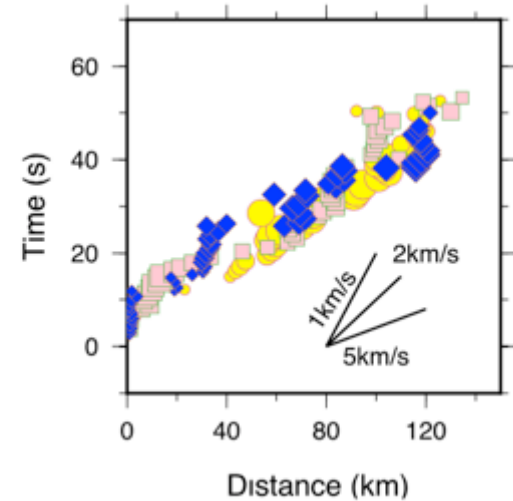
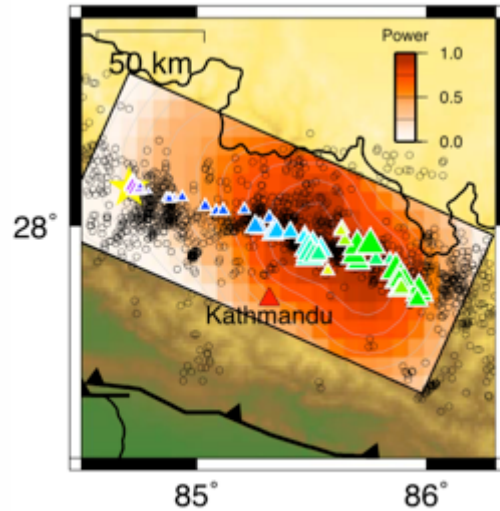
June 30 - July 4, 2019

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# Earthquake kinematics



Ide, 1997



Meng et al. (2016)

How to link kinematics and dynamics of earthquakes?

Can we predict the earthquake size based on earthquake dynamics theory?

# Outline

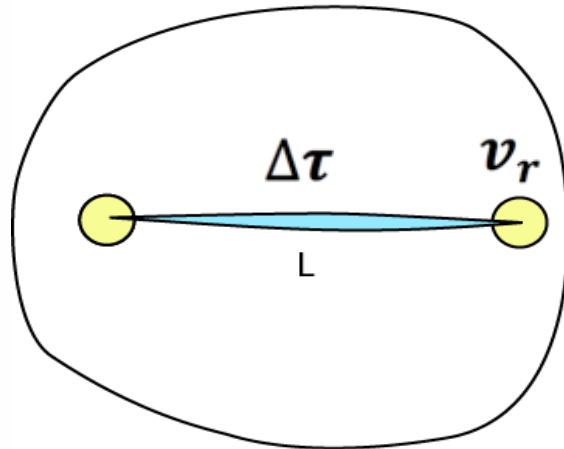
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- **Motivations**
  - Model (theory and simulations)
  - Implications
  - Ongoing work
-

# Linear elastic fracture mechanics

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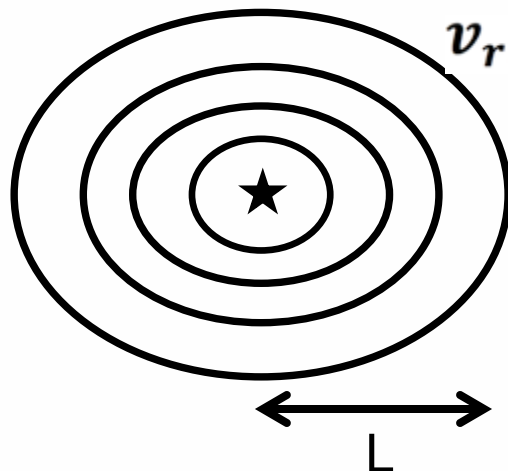
2D



For crack-like ruptures in 2D and 3D (unbounded):

$$G_c = g(v) \frac{\Delta\tau^2 L}{2\mu}$$

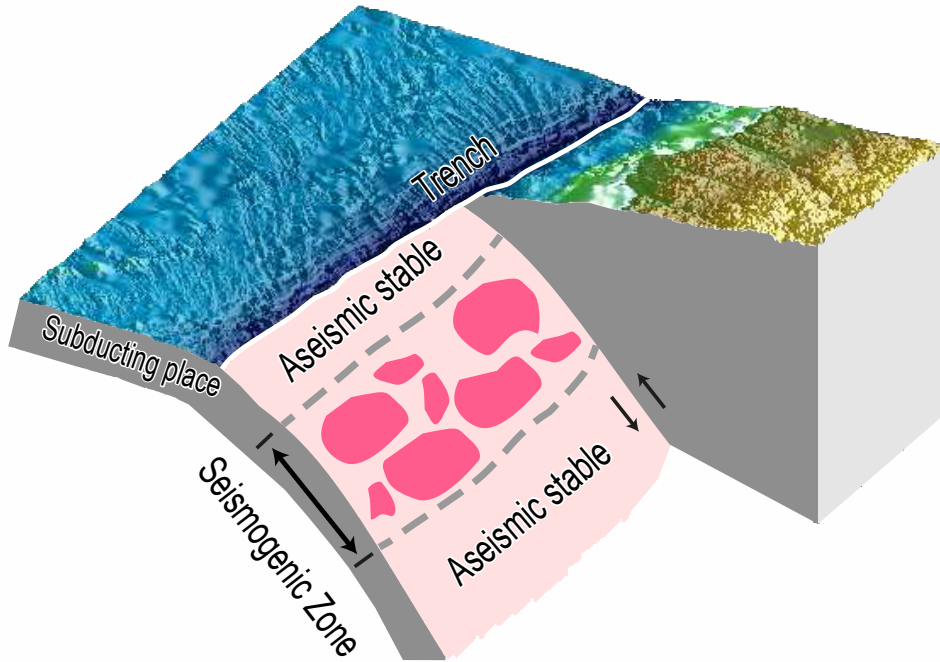
3D



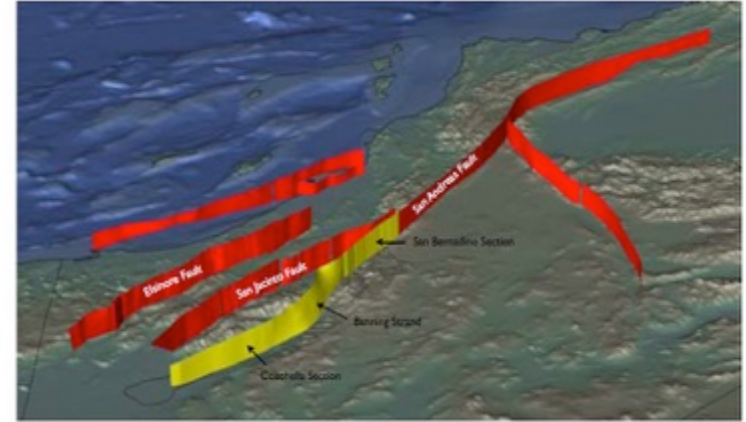
Kostrov, Freund, Andrews (60-70s)

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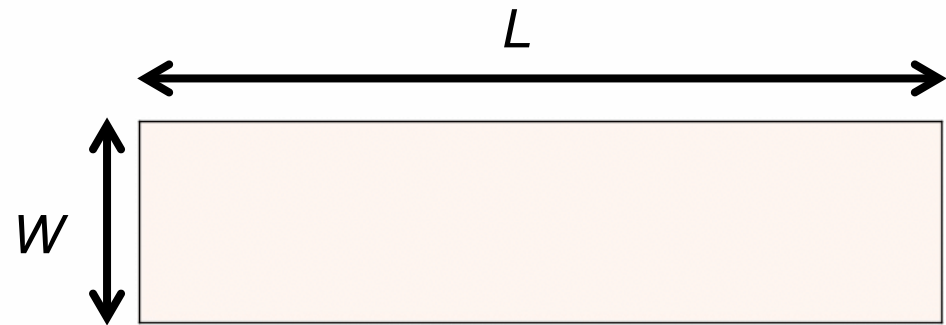
# Finite seismogenic width



Weng and Ampuero, JGR, in revision

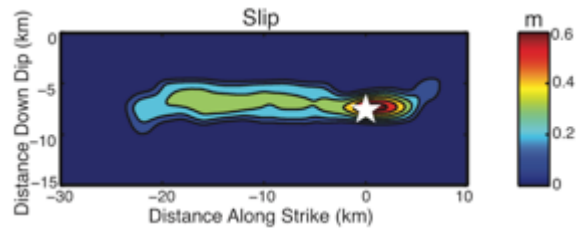


Fault and Rock Mechanics (FARM)

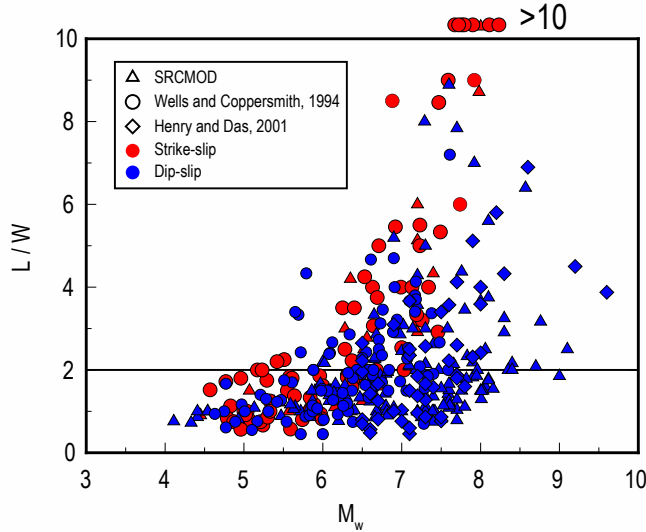


# Elongated earthquake ruptures

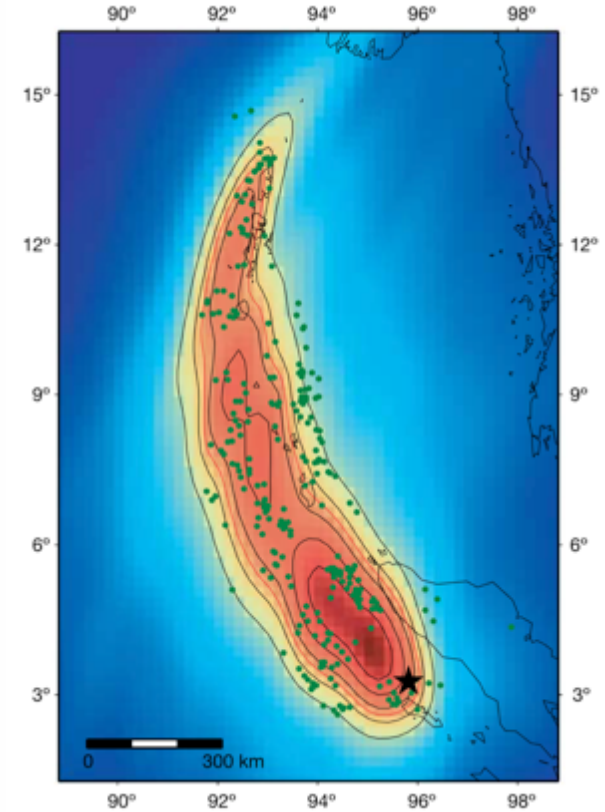
2004 Mw 6 Parkfield



Ma et al 2008



2004 Mw 9.3 Sumatra

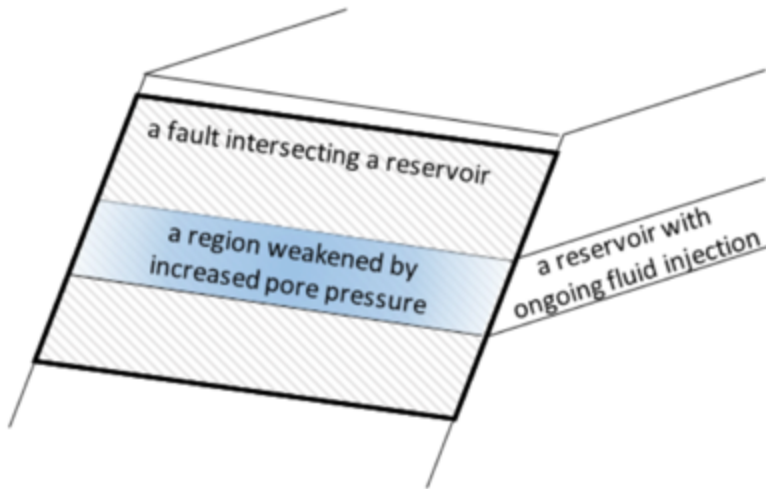


Ishii et al 2005

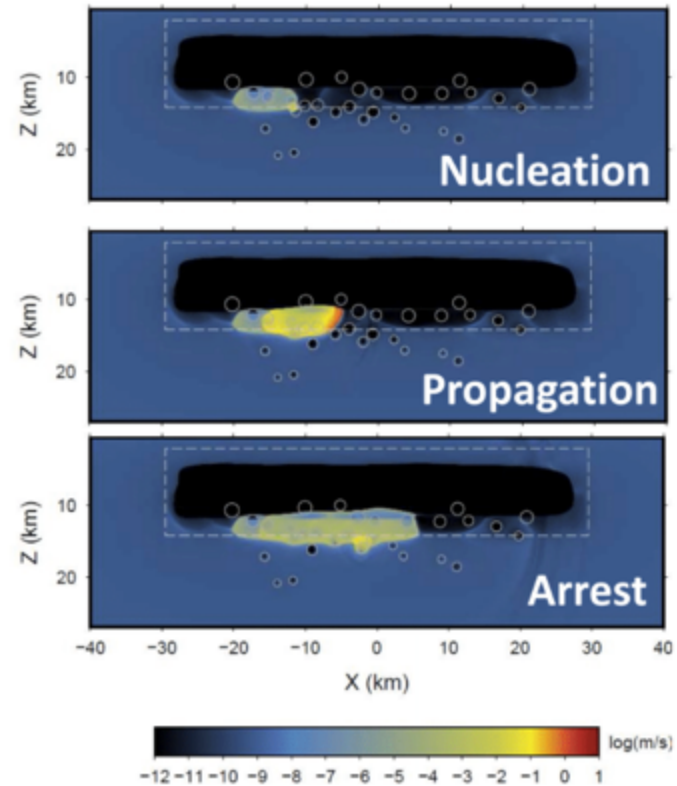
# Elongated earthquake ruptures

Rupture unzipping the lower edge of the seismogenic zone (simulation by Junle Jiang)

A fluid injection into a reservoir



Galis et al 2018



# Outline

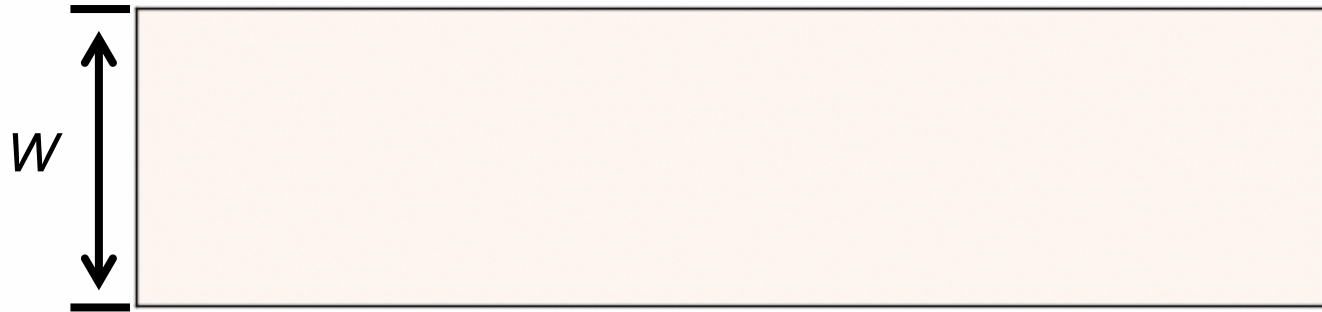
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- Motivations
  - **Model (theory and simulations)**
  - Implications
  - Ongoing work
-



# Analytical model

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## Ingredients

- Anti-plane fault in 3D full-space
- Uniform elastic properties
- Uniform fault parameters
- Uniform seismogenic width
- Steady-state speed

2.5D model

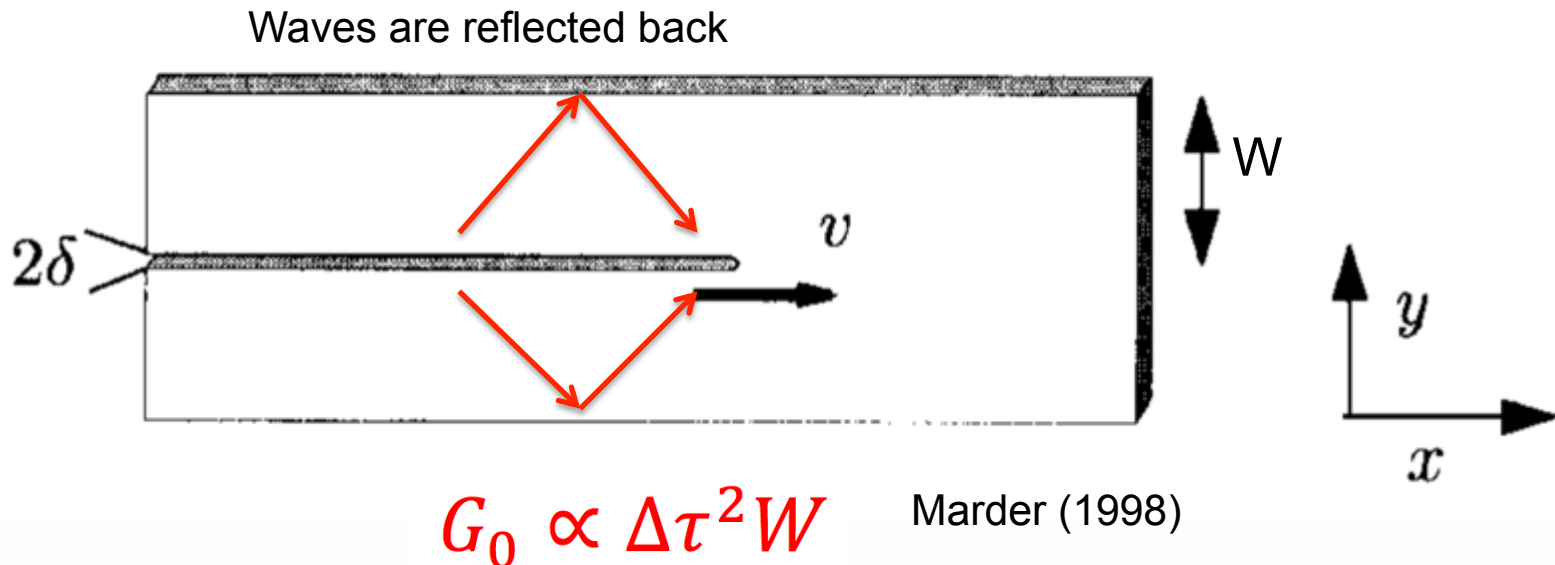
Energy release rate ( $L > W$ ):

$$G_0 = \frac{\Delta\tau^2 W}{\pi\mu}$$

Weng and Ampuero, JGR, in revision

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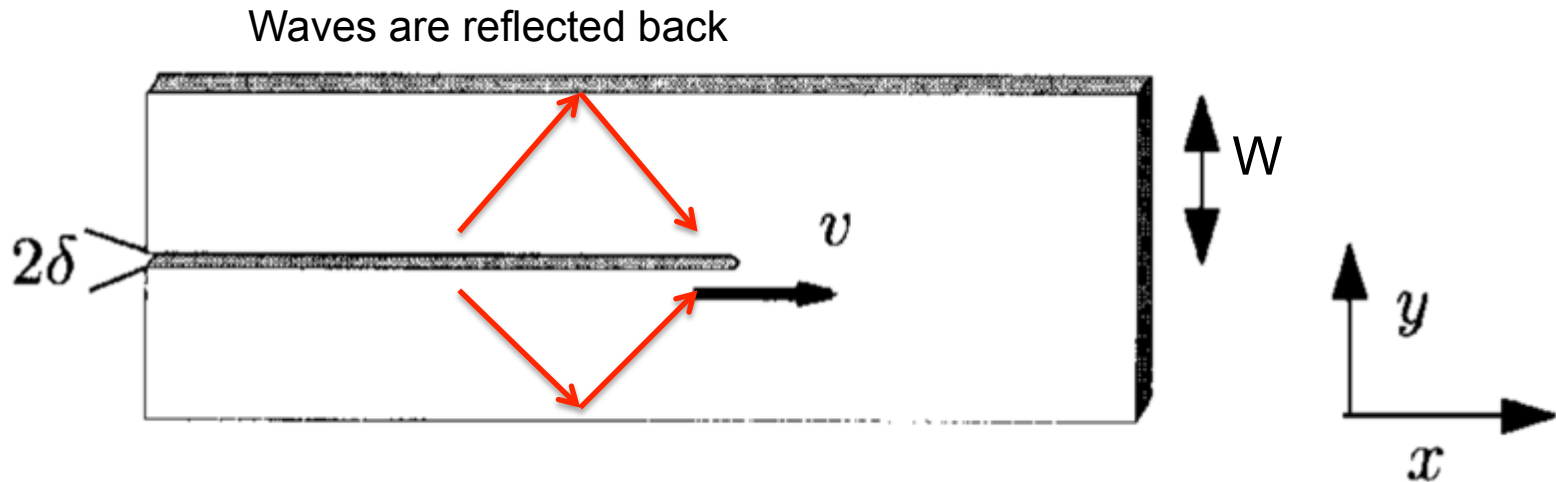
# 2D strip problem (mode I crack)



- Steady-state energy release rate is proportional to width of strip

- $$G_c = G_0 \left( 1 - \frac{\dot{v}_r W}{v_s^2} \frac{1}{\alpha_s^4} \right) \quad \alpha_s = \sqrt{1 - (v_r/v_s)^2}$$

# 2D strip problem (mode I crack)



$$G_0 \propto \Delta\tau^2 W \quad \text{Marder (1998)}$$

- Steady-state energy release rate is proportional to width of strip

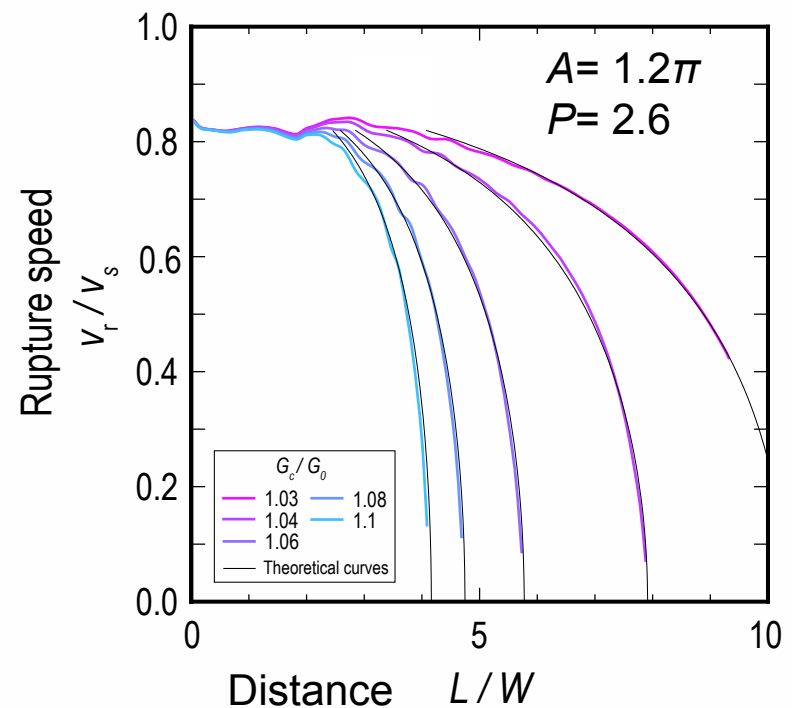
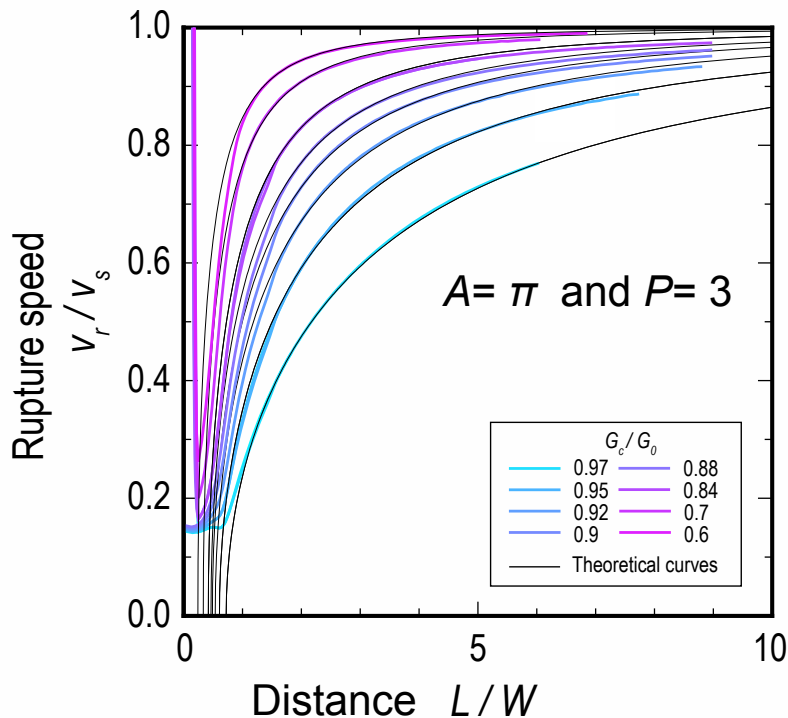
- $$G_c = G_0 \left( 1 - \frac{\dot{v}_r W}{v_s^2} \frac{1}{\alpha_s^4} \right) \quad \alpha_s = \sqrt{1 - (v_r/v_s)^2}$$

# Validation in 3D simulations

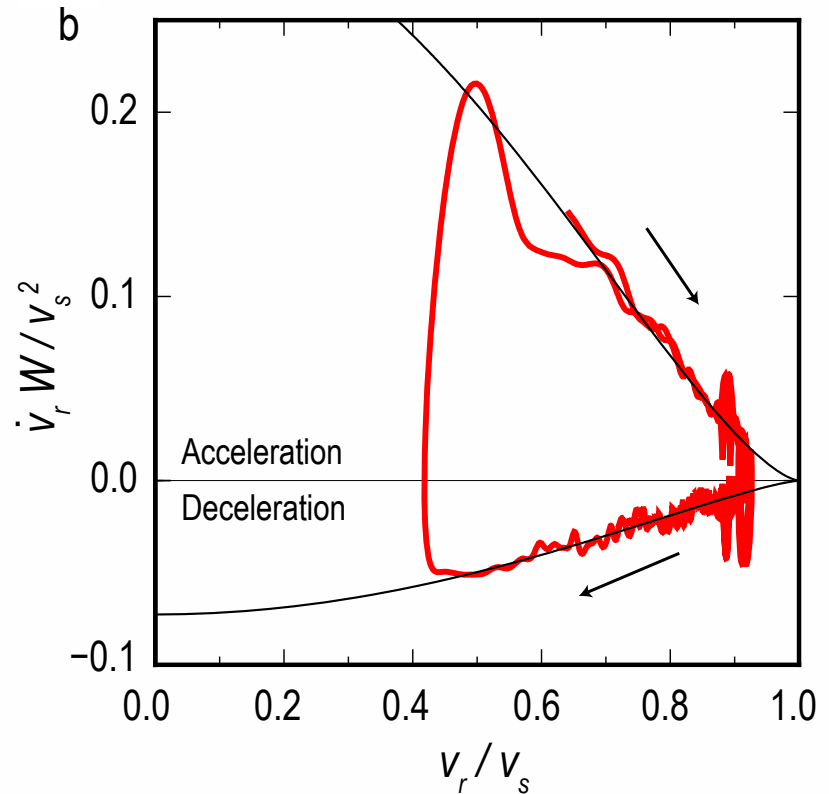
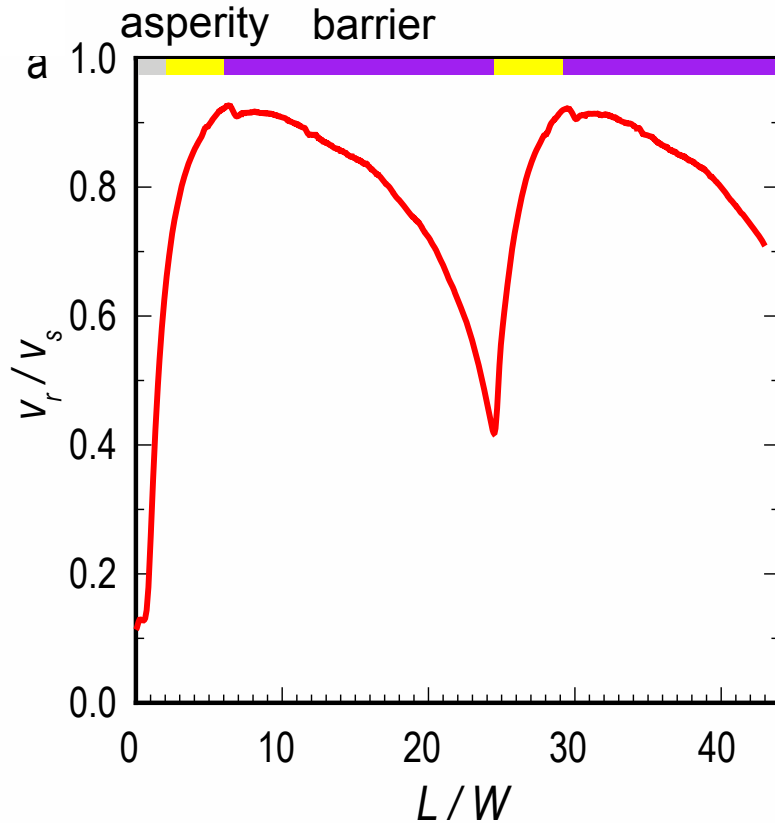
$$G_c = G_0 \left( 1 - \frac{\dot{v}_r W}{v_s^2} \frac{1}{A \alpha_s^P} \right)$$

Theoretical equation:

$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$



# “Inertial” rupture



- Rupture evolution predicted by rupture-tip-equation-of-motion
- Rupture is also “inertial”

# Outline

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- Motivations
  - Model (theory and simulations)
  - **Implications**
    - Final earthquake size
    - Super-cycles
    - Seismicity frequency-size distr.
  - Ongoing work
-

# Determine earthquake size

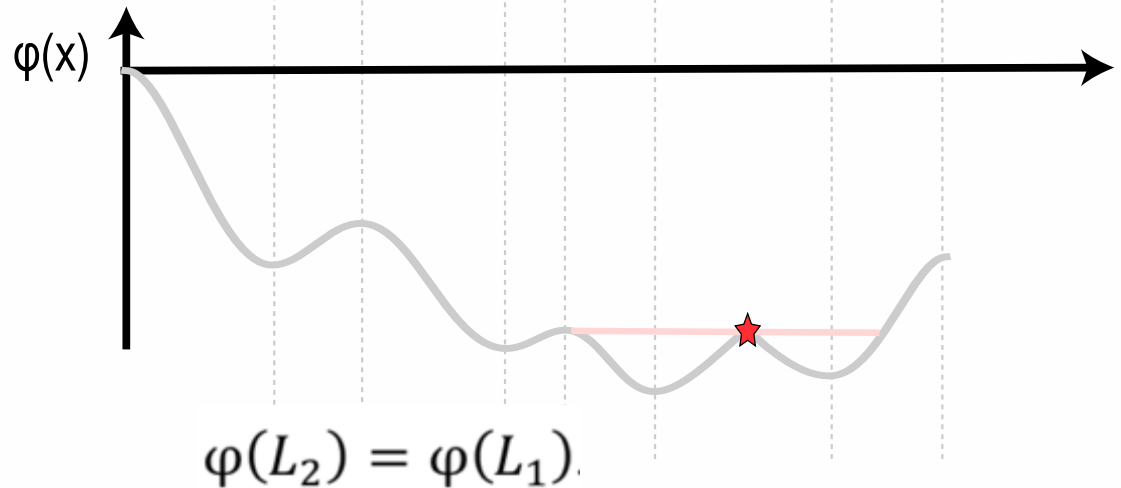
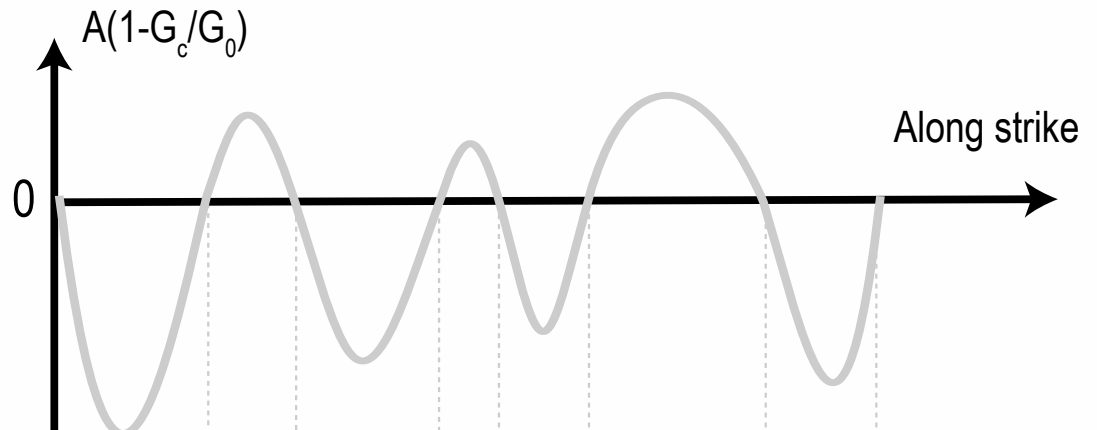
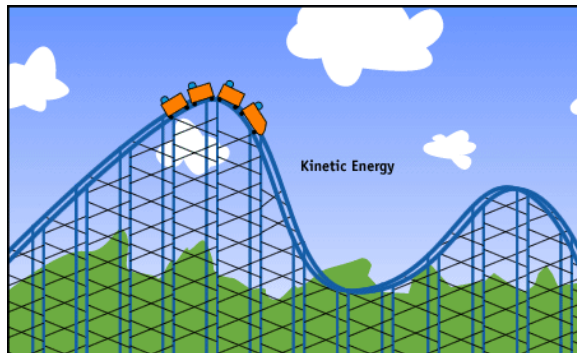
$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = A \alpha_s^P$$



$$\varphi(L) = \int_0^L A(1 - G_c/G_0) dx/W$$

Rupture potential

Gravity potential



Weng and Ampuero, JGR, in revision

# Determine earthquake size

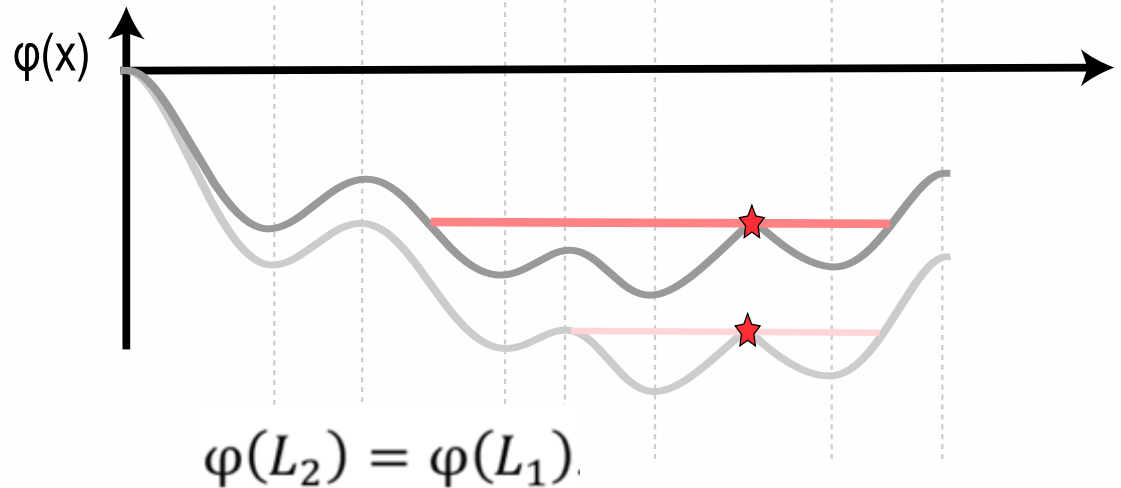
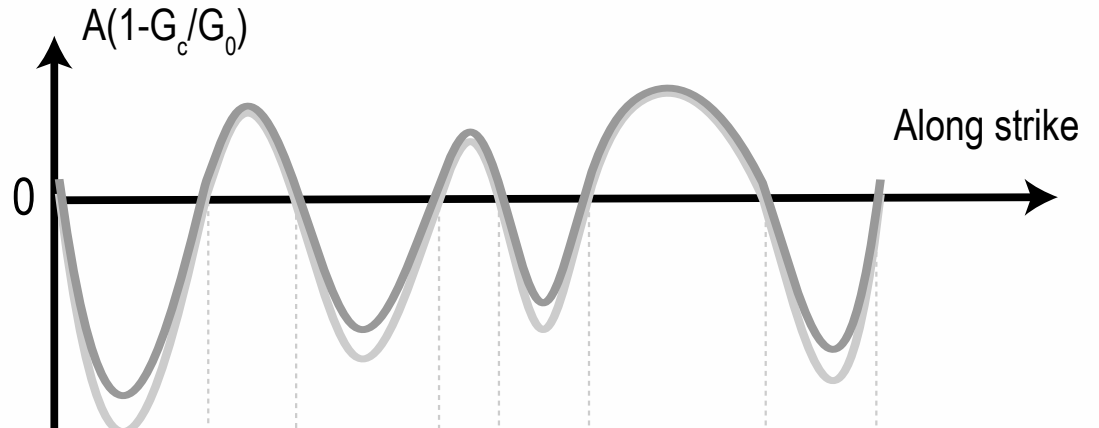
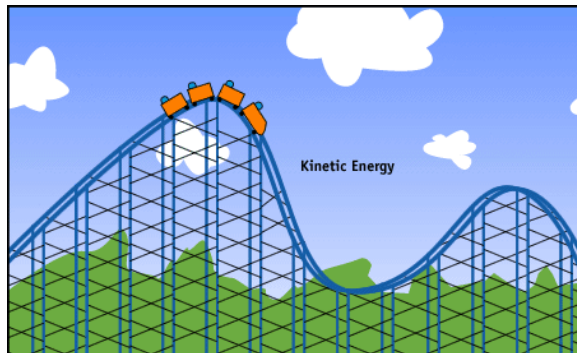
$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = A \alpha_s^P$$



$$\varphi(L) = \int_0^L A(1 - G_c/G_0) dx/W$$

Rupture potential

Gravity potential



Weng and Ampuero, JGR, in revision



# Determine earthquake size

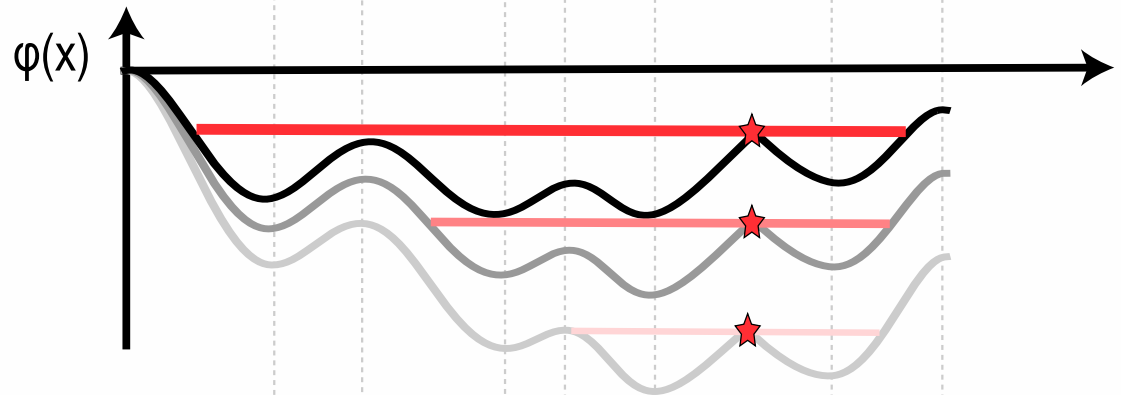
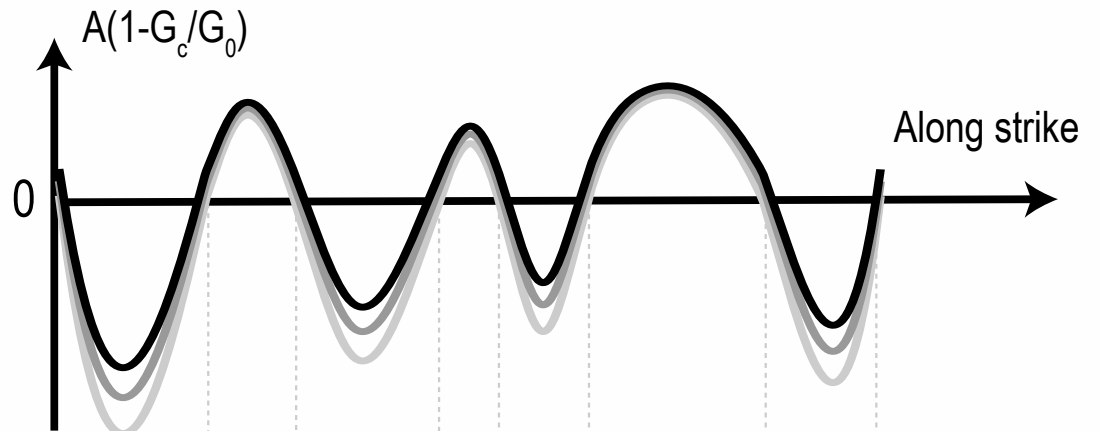
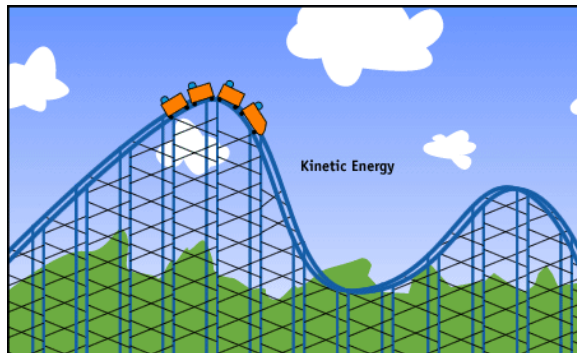
$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = A \alpha_s^P$$



$$\varphi(L) = \int_0^L A(1 - G_c/G_0) dx/W$$

Rupture potential

Gravity potential



$$\varphi(L_2) = \varphi(L_1).$$

Weng and Ampuero, JGR, in revision



# Super cycles

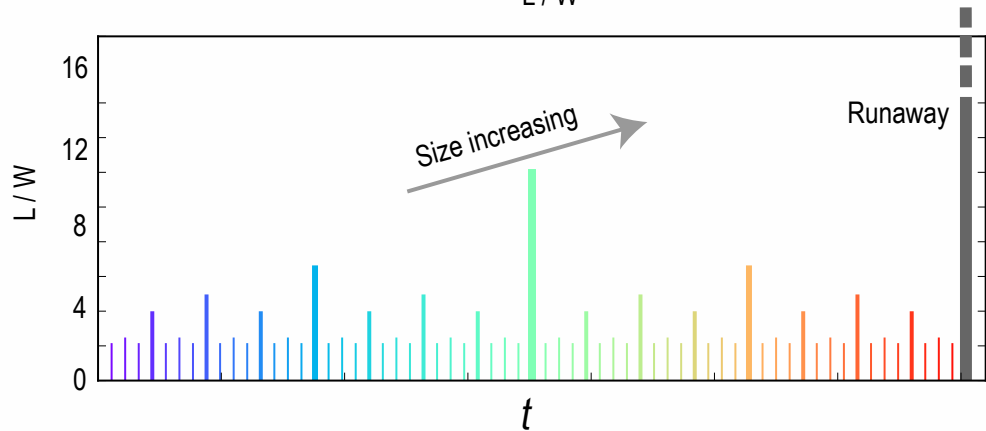
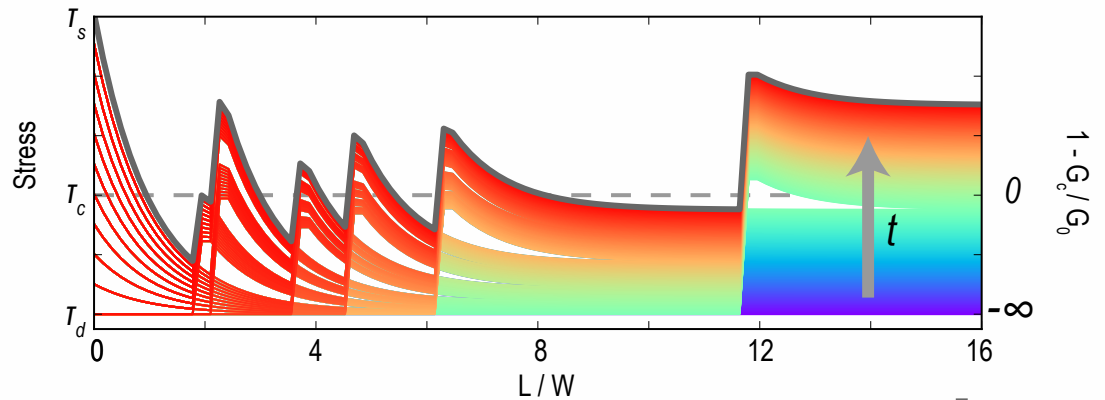


Stressing rate:

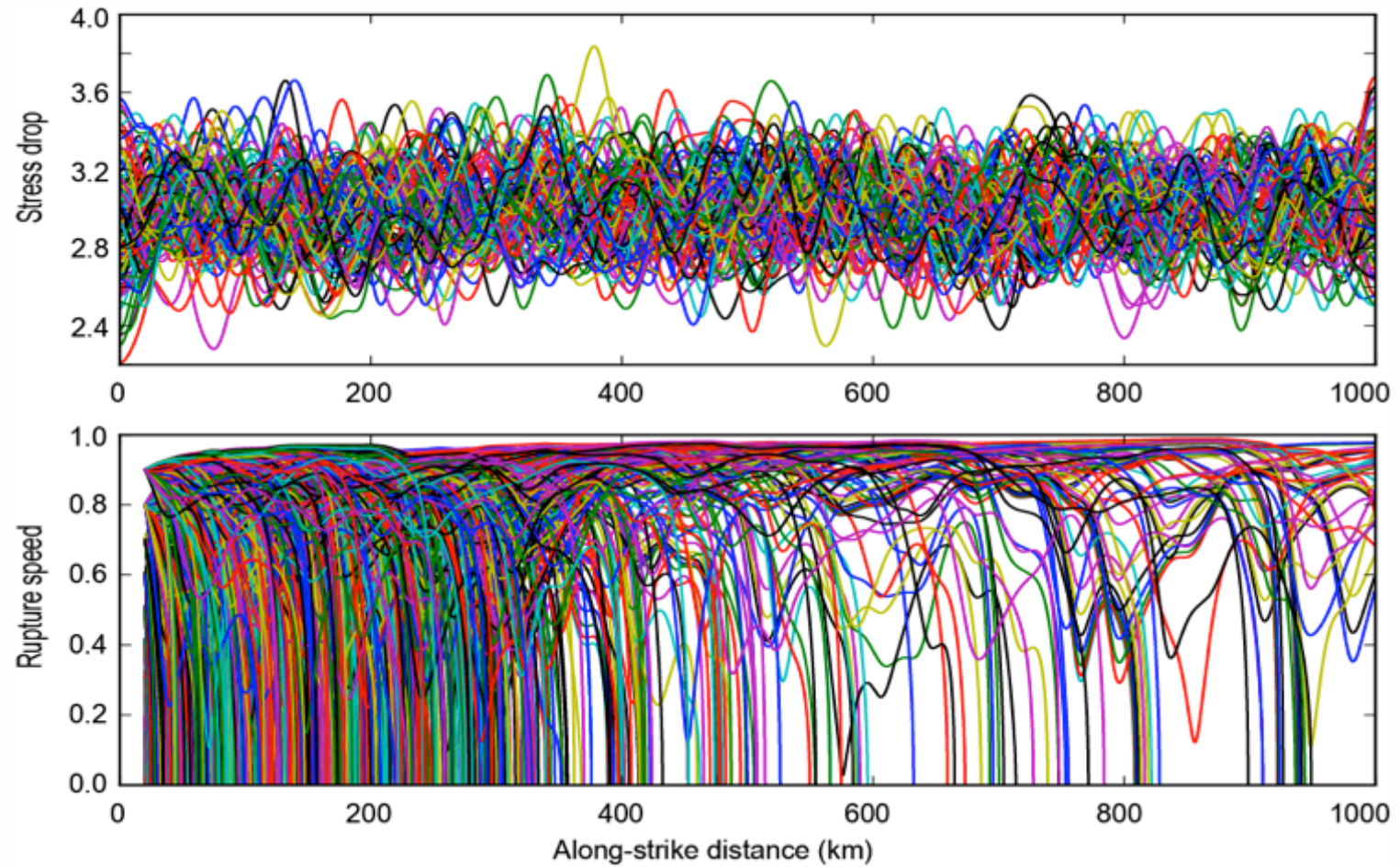
$$\dot{\tau}(L) = \gamma_l \exp(-L/W) + \gamma_b$$

Assumption:

$$G_c/G_0 = B\Delta\tau^{n-2}$$



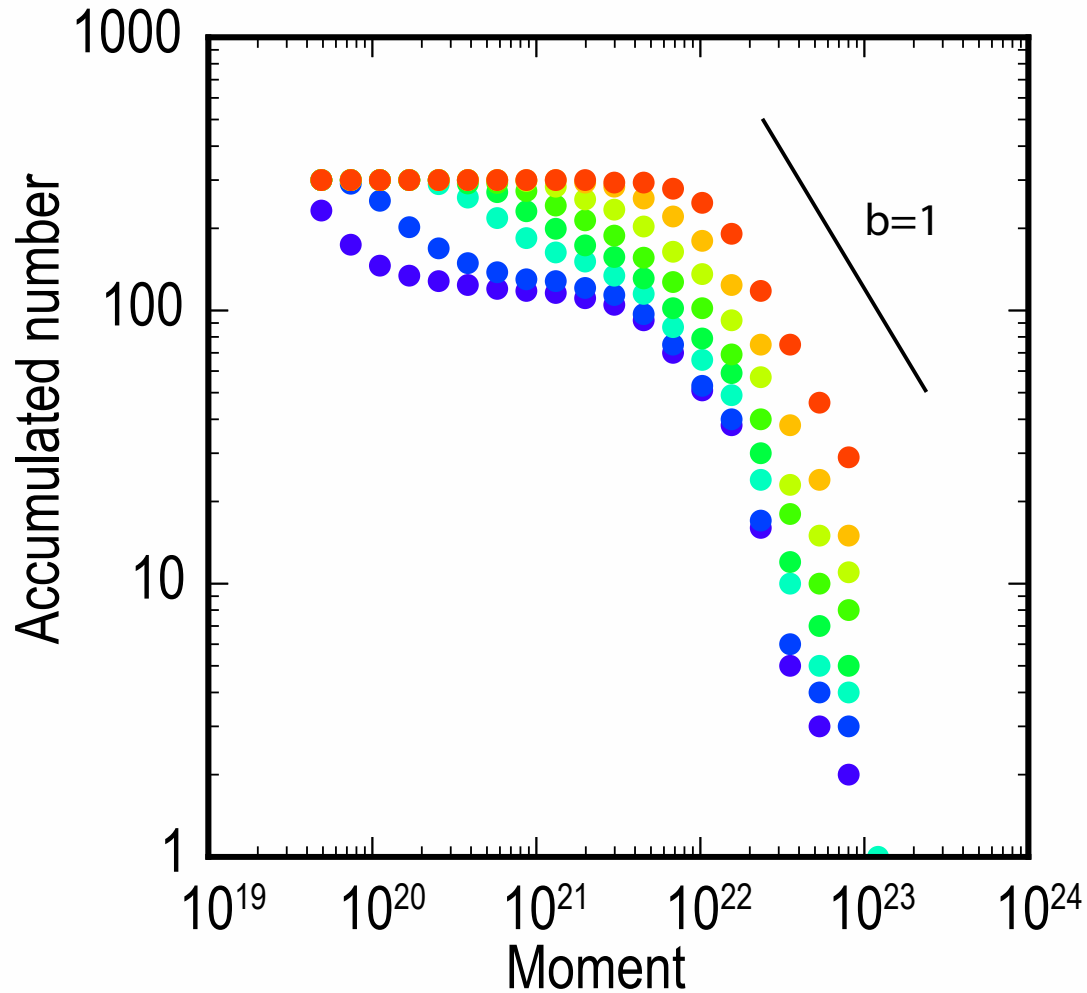
# Seismicity frequency-size distribution



Assumption:  $G_c/G_0 = B\Delta\tau^{n-2}$

# Seismicity frequency-size distribution

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# Outline

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- Motivations
  - Model (theory and simulations)
  - Implications
  - **Ongoing work: supershear**
-

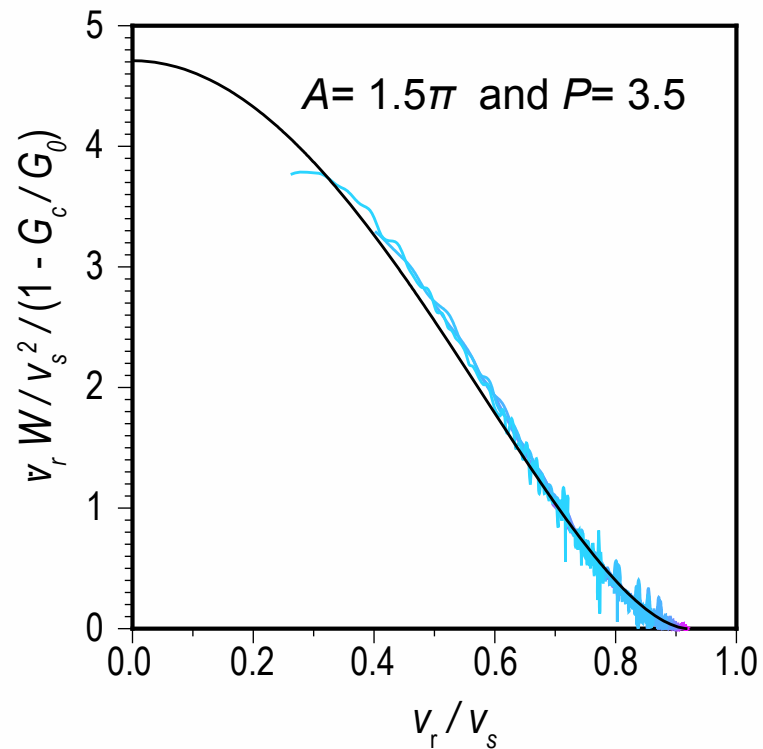
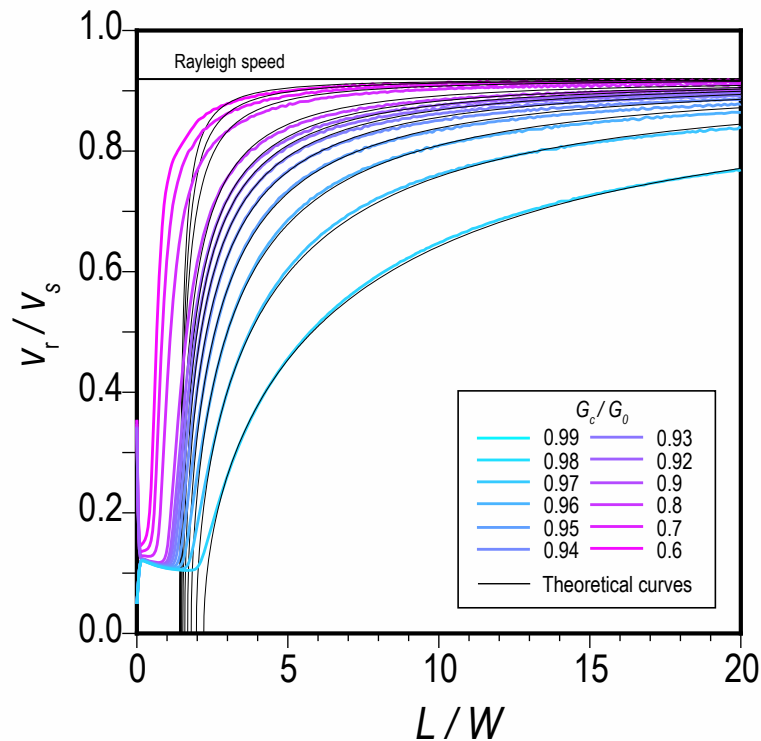


# In-plane sub-shear

$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = A \alpha_R^P$$

Theoretical equation:

$$\alpha_R = \sqrt{1 - (v_r/v_R)^2}$$



# Dynamics of supershear ruptures

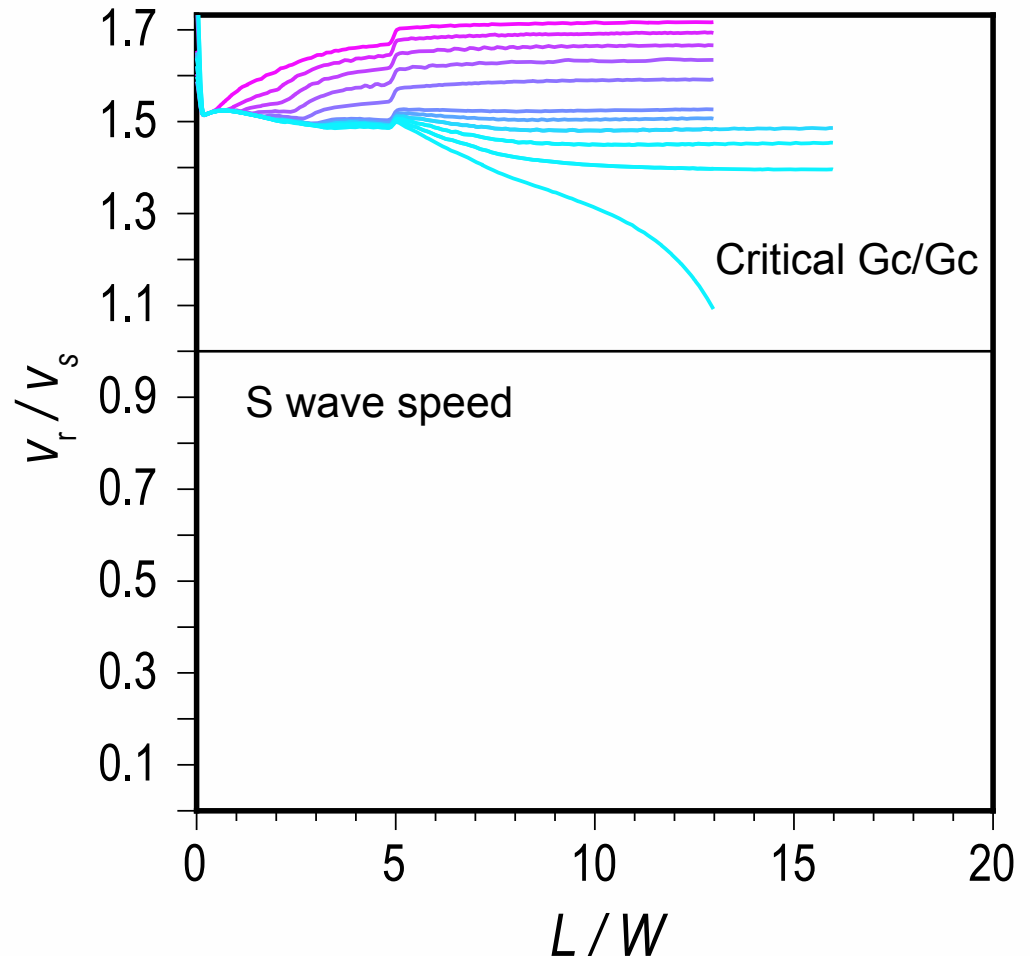
- Steady-state supershear
- $G_c/G_0$  controls supershear speed
- Critical value of  $G_c/G_0$  for supershear

On-going analytical work:

$$G^{sup} = g(v_r)G_0 \left(\frac{\Lambda}{W}\right)^{q(v_r)}$$

Weng and Ampuero, In prep.

## 3D numerical simulations





# Conclusion

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- A new rupture-tip-equation-of-motion for elongated ruptures elucidates how the evolution of rupture speed of large earthquakes (large aspect ratio) depends on fault strength and stress.
- This theoretical equation has important implications for evaluating how final earthquake size depends on fault stress and strength.
- The seismogenic width also plays significant effects on dynamics of supershear ruptures.

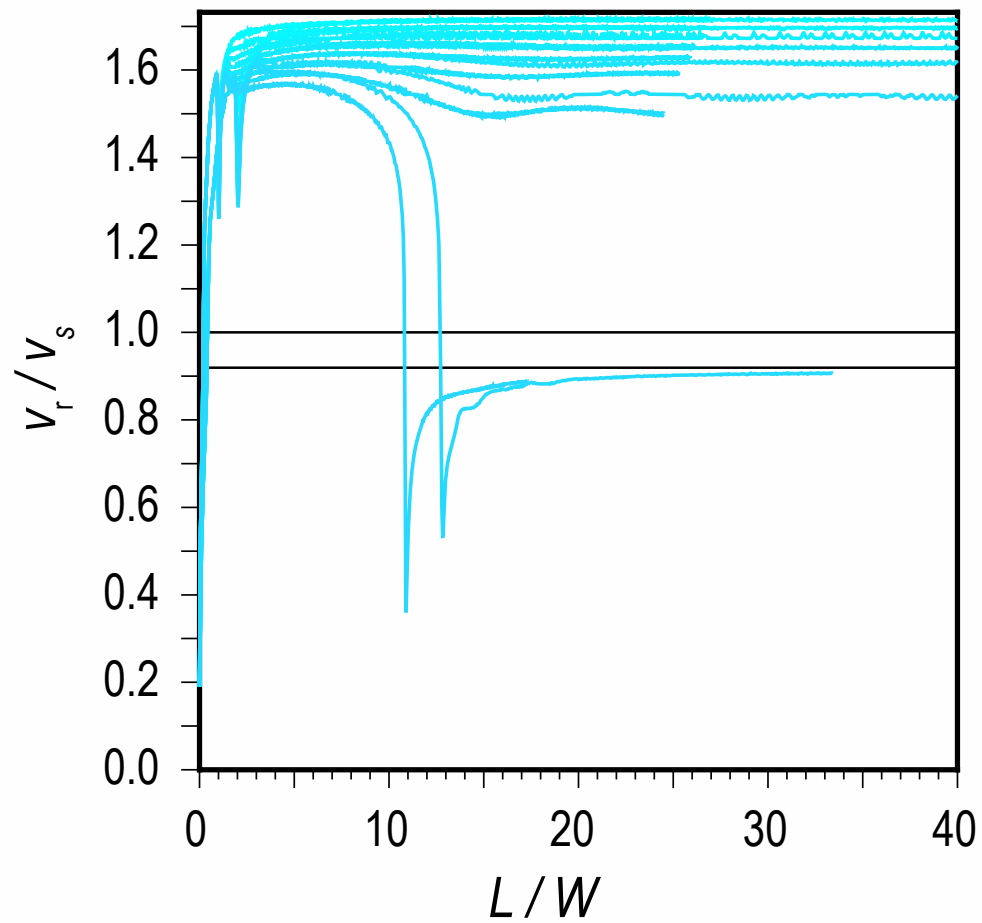
The manuscript can be download from EarthArXiv:

[eartharxiv.org/9yq8n/](https://eartharxiv.org/9yq8n/)

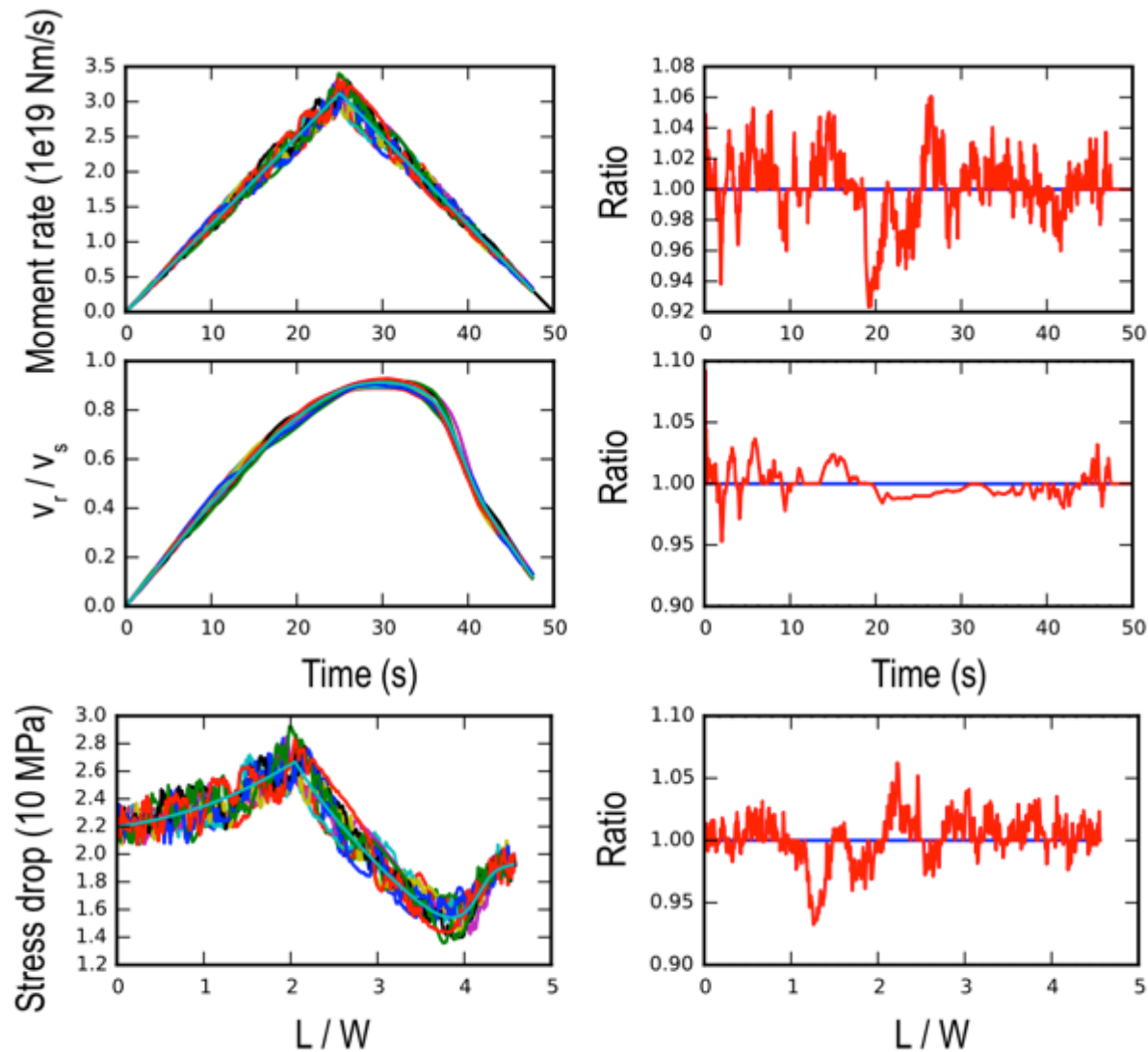
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# Information from source time function



# Analytical model

$$\sigma_{ij,j} = \rho \ddot{u}_i \quad (3 \text{ equations})$$



Reduce to 1 equation

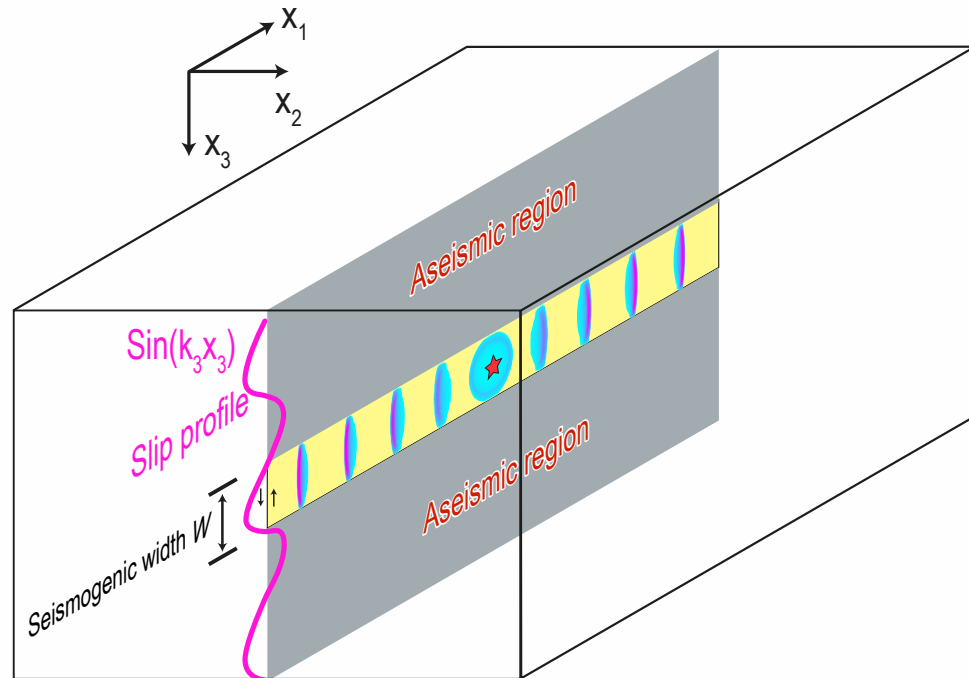
$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} = \frac{1}{v_s^2} \frac{\partial^2 u}{\partial t^2}$$

Slip approximation

$$u(x_1, x_2, x_3) = u(x_1, x_2, t) e^{ik_3 x_3}$$

$$k_3 = \pi/W$$

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} - k_3^2 u = \frac{1}{v_s^2} \frac{\partial^2 u}{\partial t^2}$$

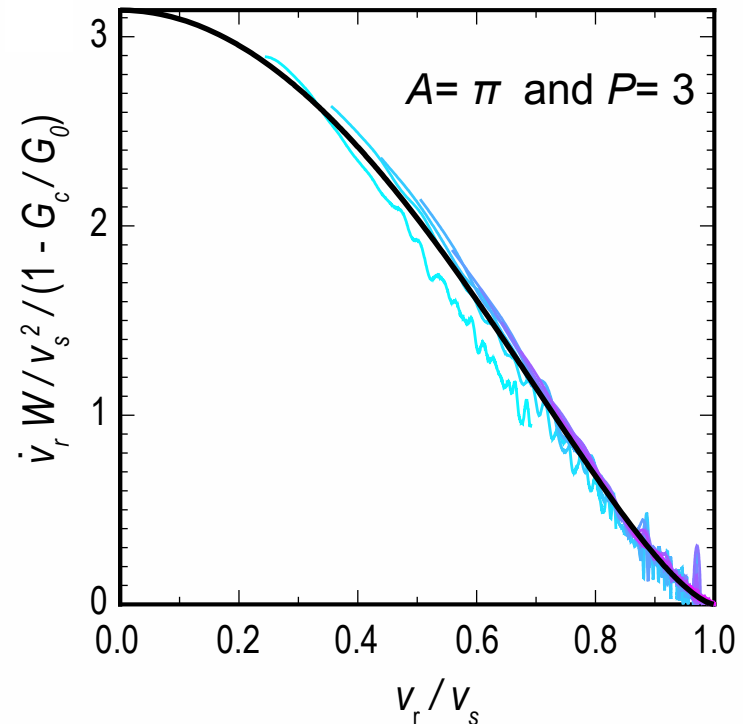
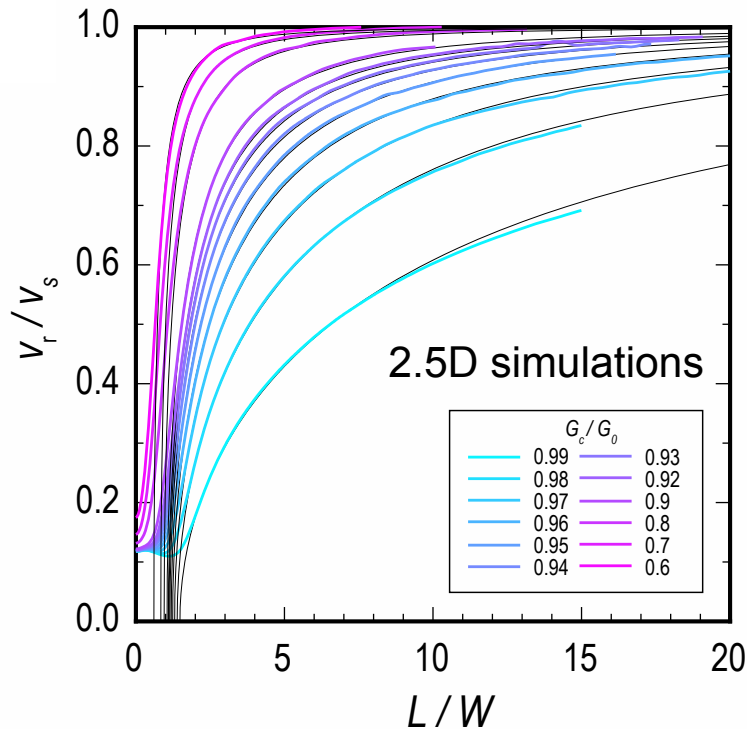


# Rupture acceleration

- $G_0 > G_c \rightarrow$  ruptures accelerate  $\uparrow$
- $G_c/G_0$  plays an important role in controlling rupture speed

$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = \pi \alpha_s^3$$

$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$

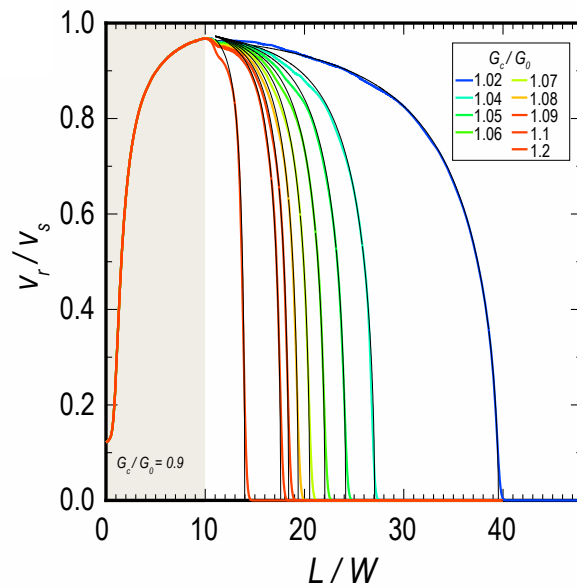
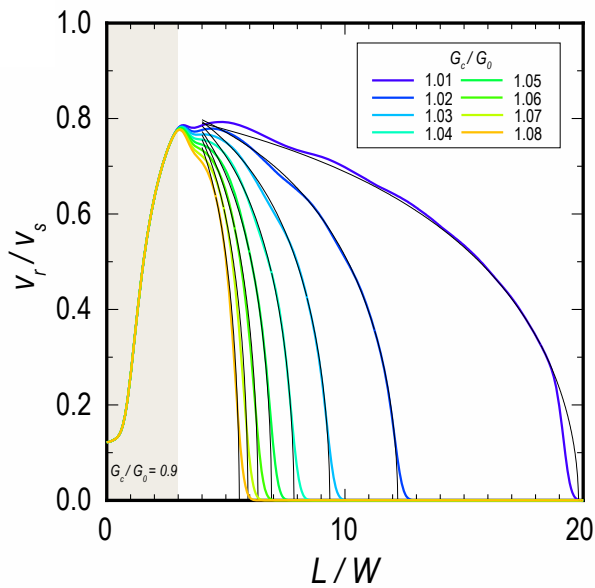


# Rupture deceleration

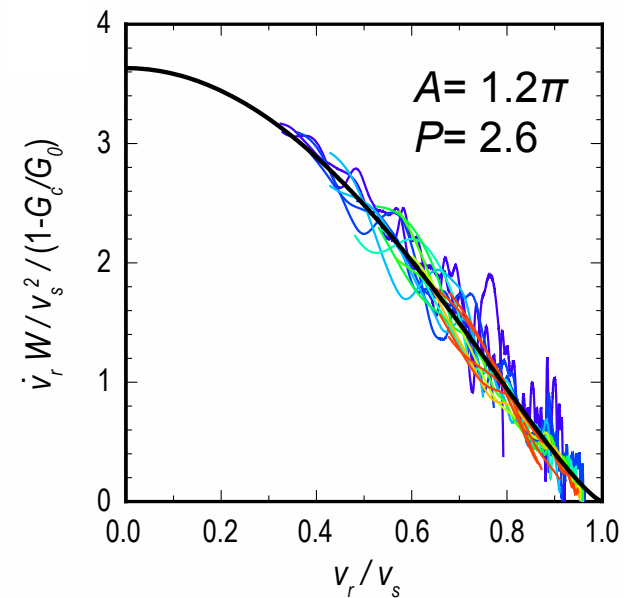
- $G_0 < G_c \rightarrow$  ruptures decelerate  $\downarrow$
- Starting speed also plays a role
- Larger rupture speed lead to longer distance

$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = 1.2\pi \alpha_s^{2.6}$$

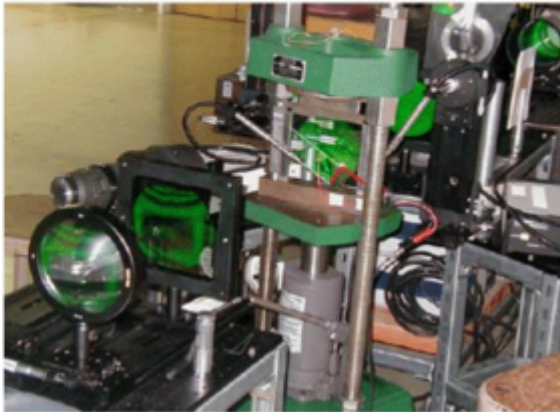
$$\alpha_s = \sqrt{1 - (v_r/v_s)^2}$$



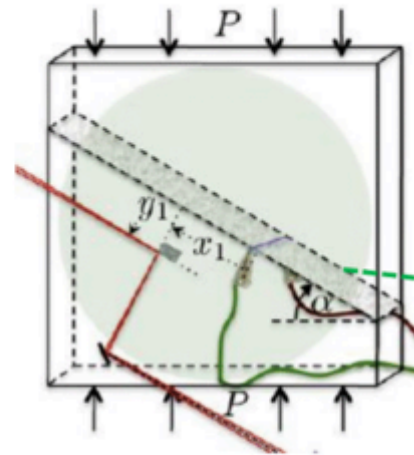
2.5D simulations



# Elongated ruptures in the lab



Laboratory earthquake experiment



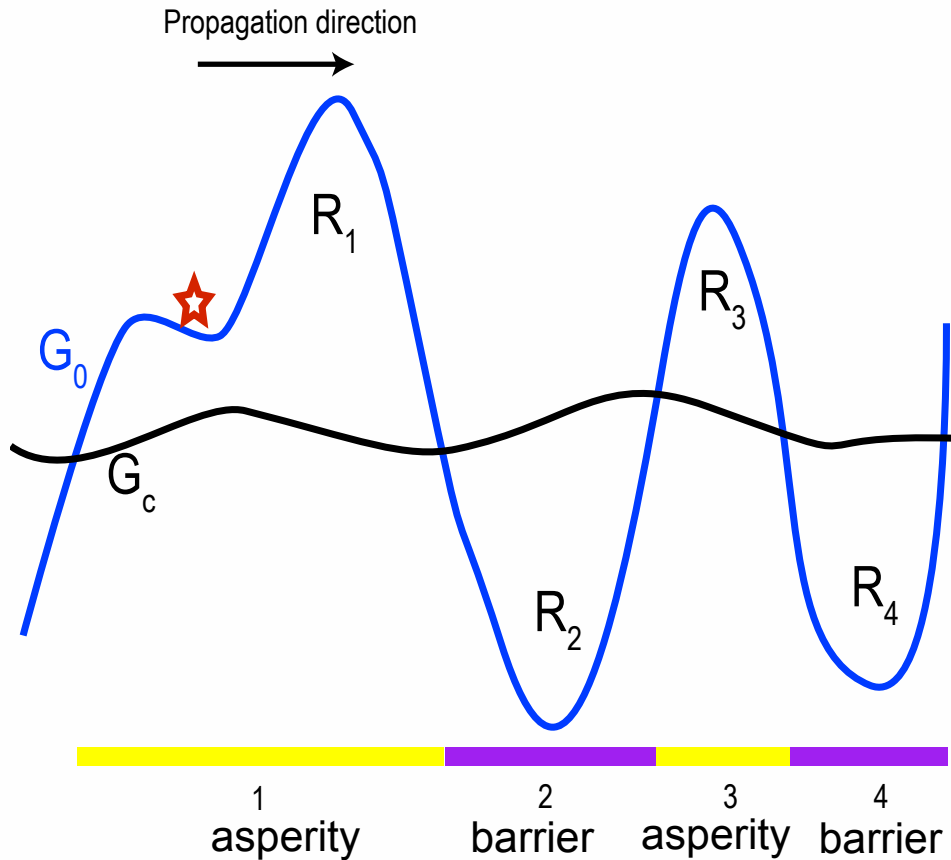
Mello et al (2014)  
in Rosakis lab (Caltech)



$$G_0 \approx \frac{\Delta\tau^2 W}{\pi\mu}$$



# Rupture potential



$$\frac{\dot{v}_r W}{v_s^2 (1 - G_c/G_0)} = A \alpha_s^P$$



$$\frac{v_r dv_r}{v_s^2 \alpha_s^P} = A(1 - G_c/G_0) dx/W$$

“Kinetic” energy? ↓ “Potential” energy?

$$\frac{1}{P-2} (\alpha_s^{2-P} - 1) |_{v_{r1}}^{v_{r2}} = \int_{L_1}^{L_2} A(1 - G_c/G_0) dx/W$$



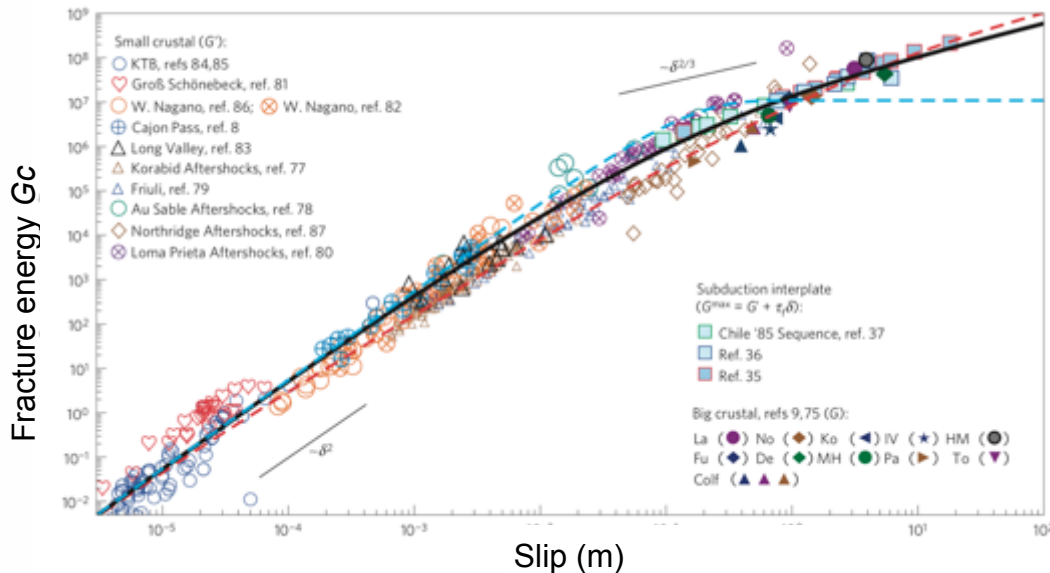
Rupture potential

$$\varphi(L) = \int_0^L A(1 - G_c/G_0) dx/W$$

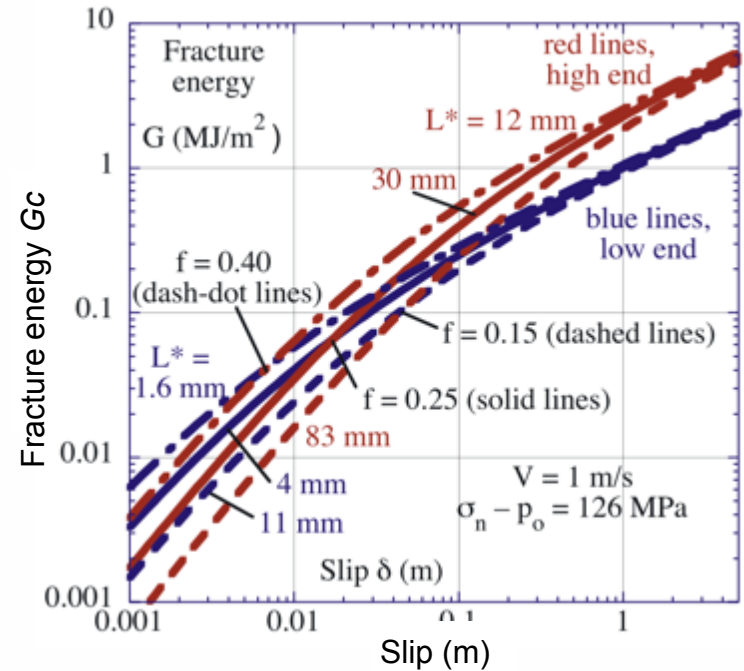
# Fracture energy on fault

Fracture energy is a function of final slip  $D(x)$ ?

For bounded fault  $D(x) = \gamma \hat{W} \Delta\tau(x) / \mu$  then  $G_c \propto \Delta\tau^n$  ?

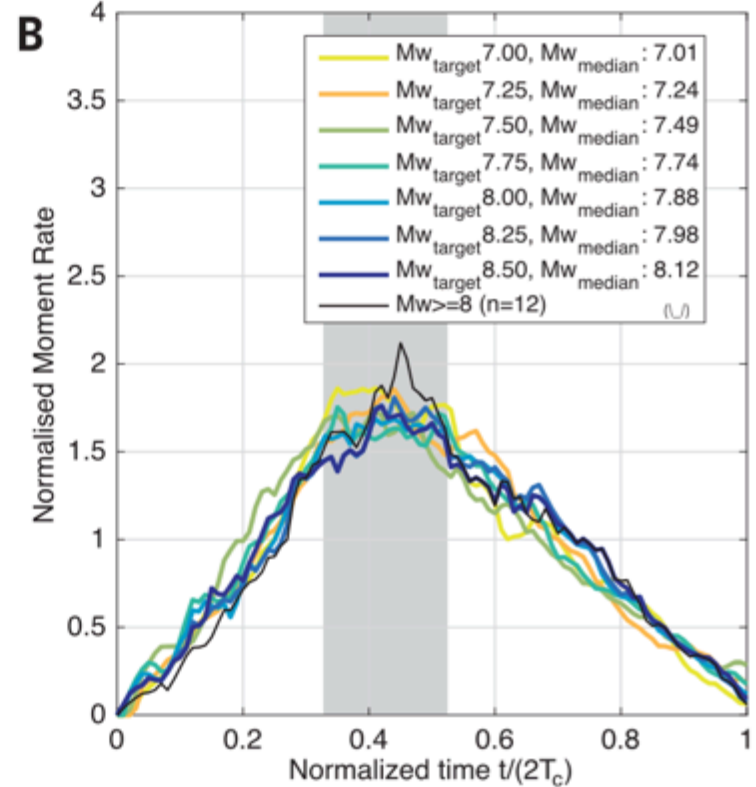
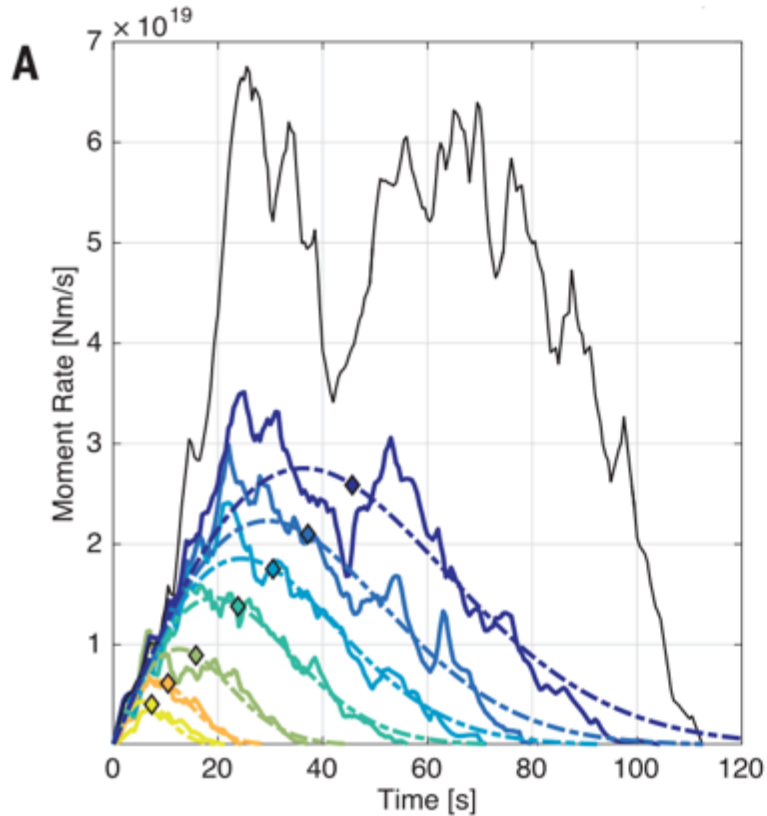


Viesca and Garagash (2015)



Rice (2006)

# Source time function of earthquakes

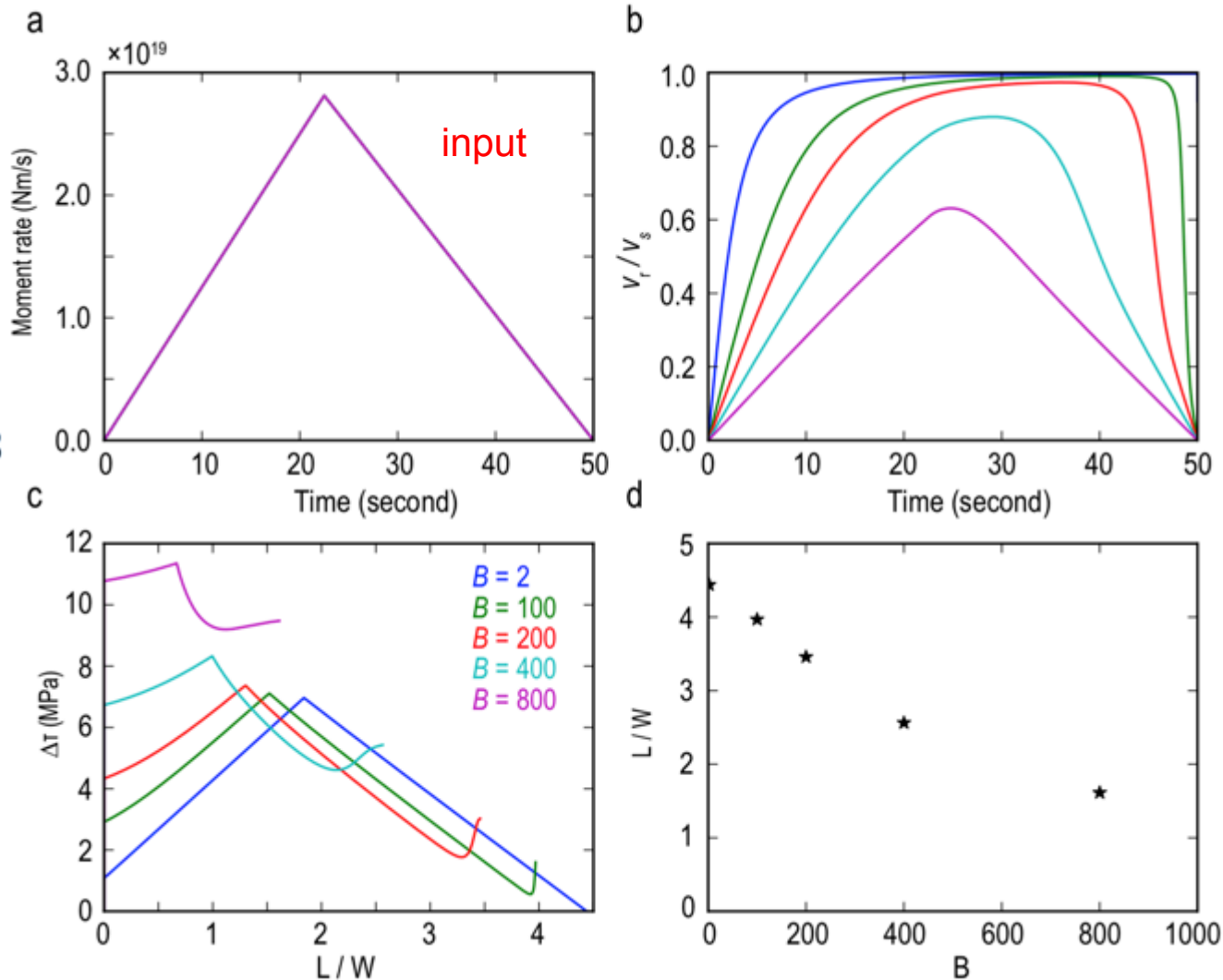


General pattern of earthquake – triangular

Meier et al 2017

What is the intrinsic physics ?

# Constraints from STF



$$G_c = B \Delta \tau^{2/3}$$

$$v_{r0} = 0$$

Assuming  $n=2/3$ ,  $\gamma=1$ , and  $v_r(0)=0$