Modeling Earthquake Dynamic Rupture with Hybrid Finite Element-Spectral Boundary Integral Method

Prof. Ahmed Elbanna

Xiao Ma, Mohamed Abdelmeguid and Ahmed Elbanna

Dept. of Civil and Environmental Engineering
University of Illinois at Urbana-Champaign

Mohamed Abdelmeguid

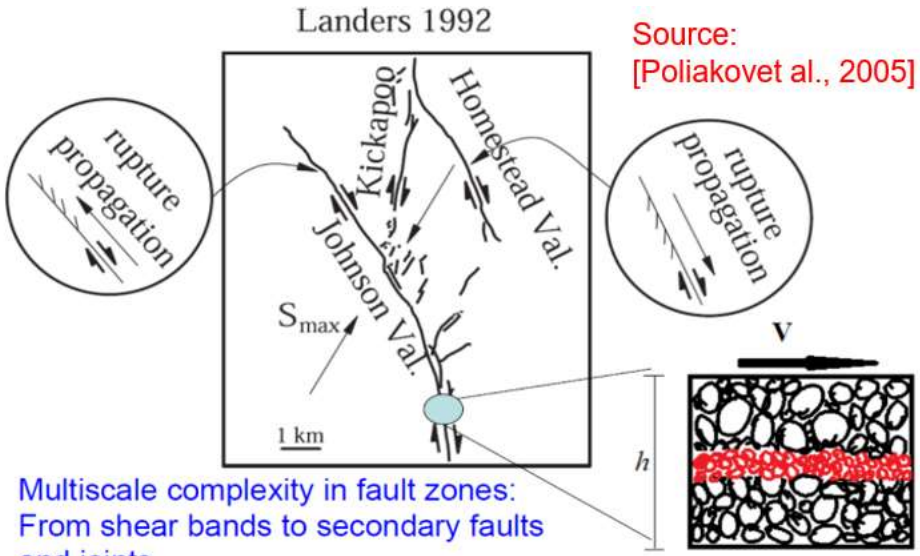
ILLINOIS

Motivation

- Fault zone has multiscale spatial-temporal complexity

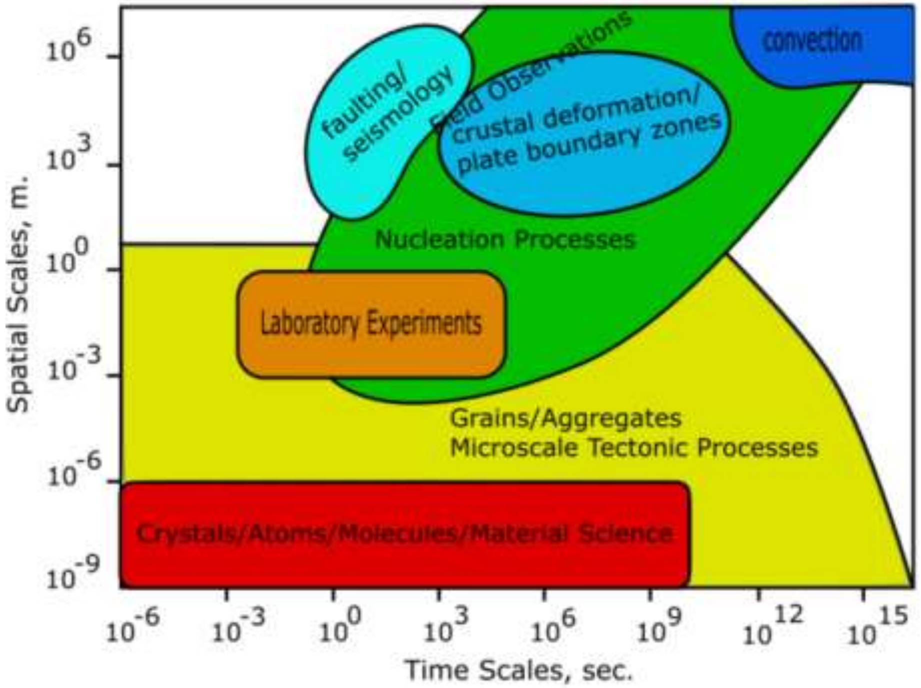
  - Spatial scales: microns to km
  - Temporal scales: fraction of seconds to years

- The multiscale feature is a challenge for numerical method.



Source:
[Poliakovet al., 2005]

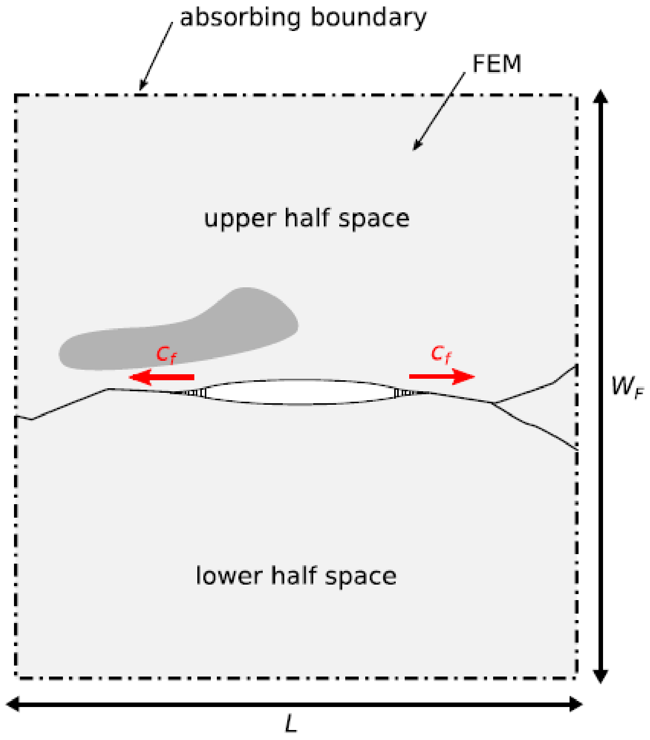
Multiscale complexity in fault zones:
From shear bands to secondary faults
and joints



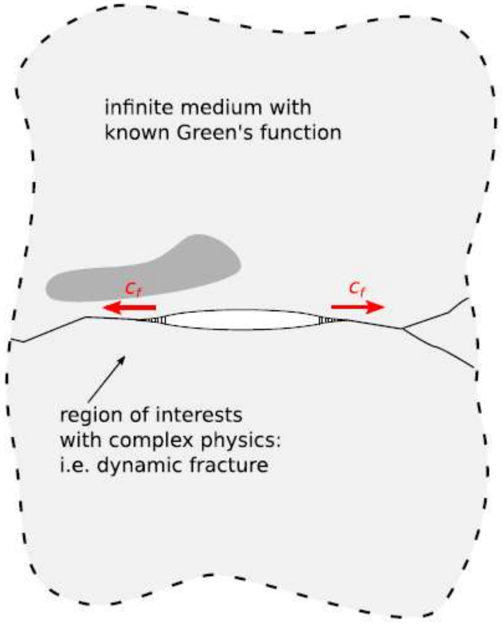
Fault zone multiscale spatial-temporal complexity

A Hybrid Finite Element-Spectral Boundary Integral approach

- Two major numerical schemes for solving earthquake problem:



Domain Based Method

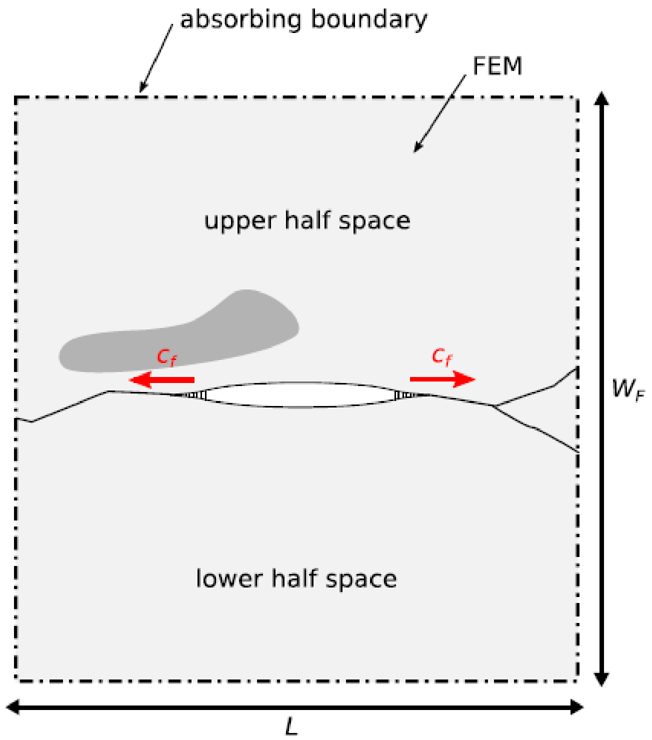


Boundary Based Method

Ma, X., Hajarolasvadi, S., Albertini, G., Kammer, D., Elbanna, A. “A hybrid finite element-spectral boundary integral approach: Applications to dynamic rupture modeling in unbounded domains”, *International Journal for Numerical and Analytical Methods in Geomechanics*, 2018.

A Hybrid Finite Element-Spectral Boundary Integral approach

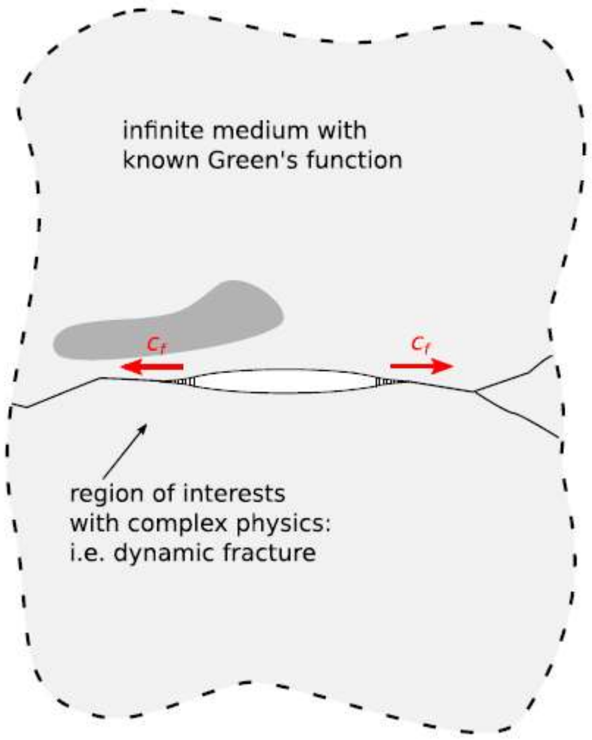
- Domain Based Method:
  - Finite Element method (FEM), Finite difference method (FDM), Finite Volume method (FV), Discontinuous Galerkin (DG), XFEM …
  - Advantage: Flexible to handle nonlinearities/heterogeneities
  - Disadvantage: Computationally demanding



Domain Based Method

A Hybrid Finite Element-Spectral Boundary Integral approach

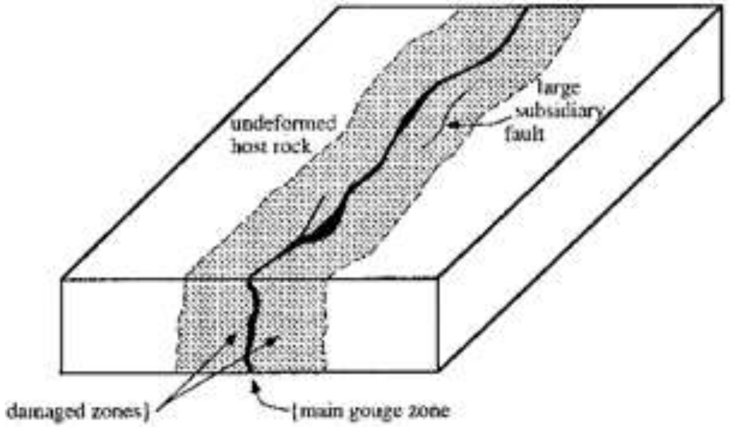
- Boundary Based Method:
  - Boundary integral Method, Spectral boundary integral method
  - Advantage: Computationally efficient
  - Disadvantage: Limited to linear-elastic homogeneous bulk



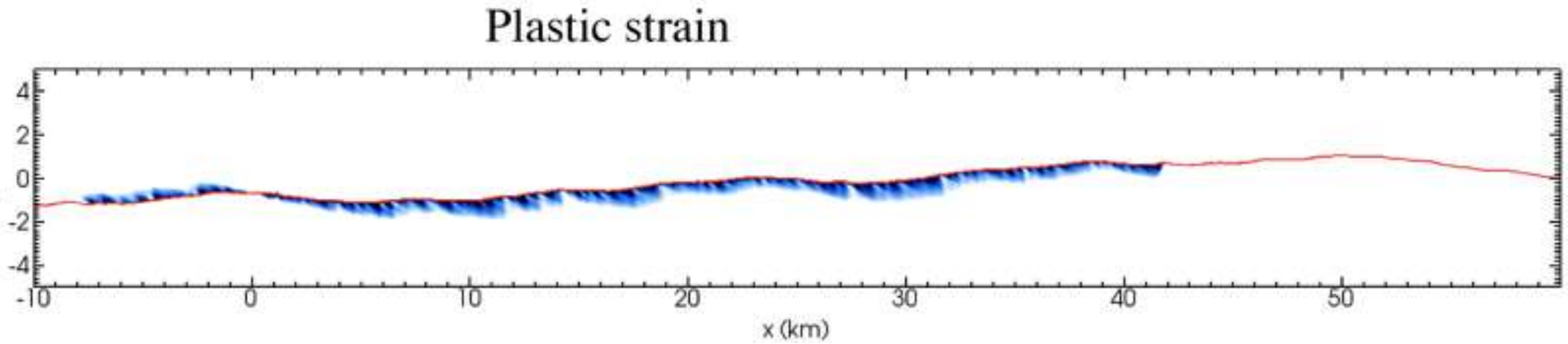
Boundary Based Method

Motivation

- Many important examples incorporated fault zone complexities have shown that:
  - Fault zone has geometric or material nonlinearities
  - The nonlinearities are relatively local.

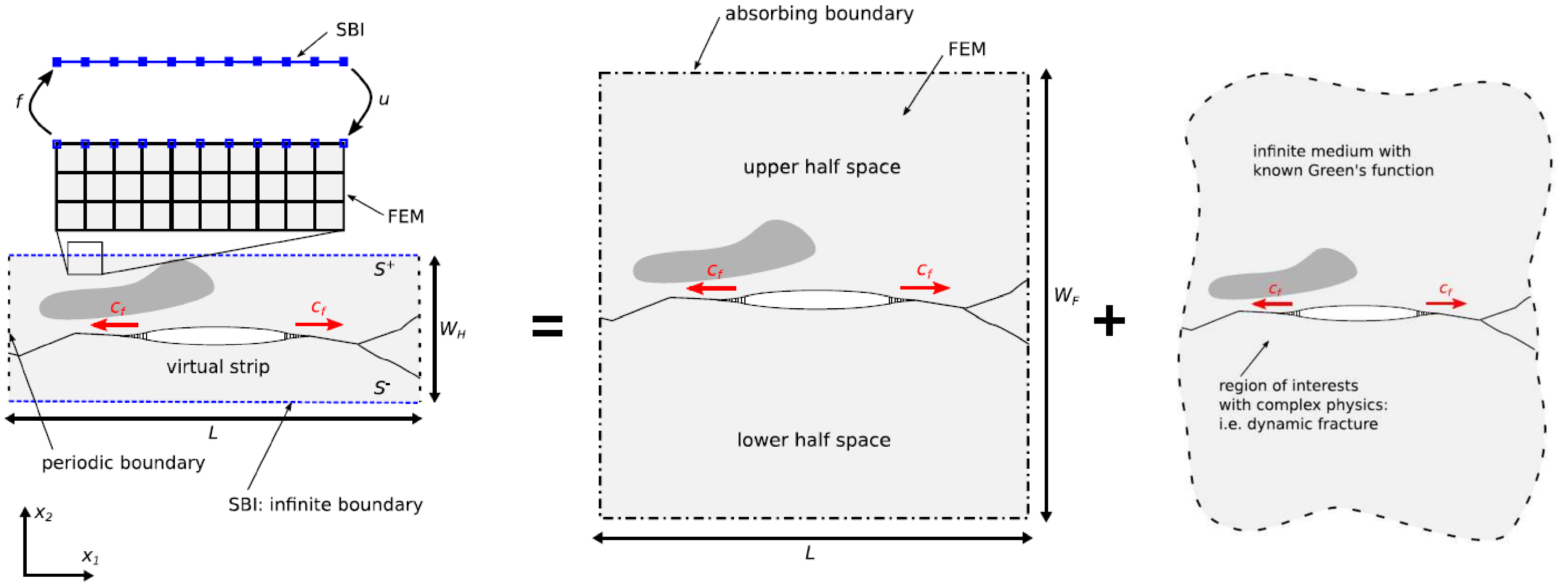


Damage and localization in fault [Chester and Logan, 1986]



Plasticity distribution with rough fault [Dumham et al, 2011]

A Hybrid Finite Element-Spectral Boundary Integral approach



Hybrid Method

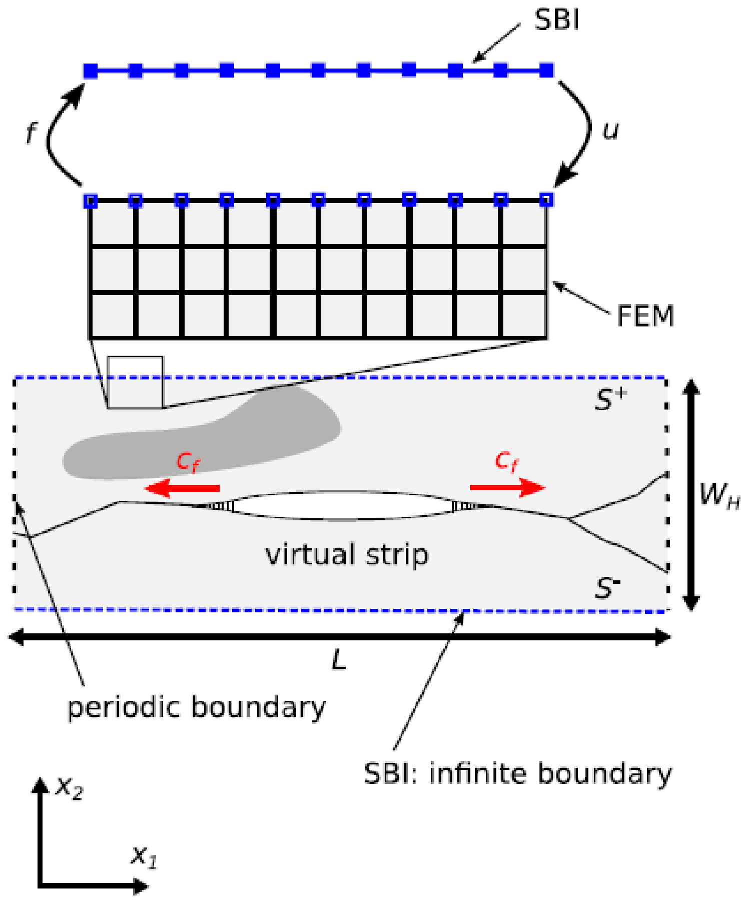
Domain Based Method

Boundary Based Method

Hybrid Method (FEM + SBI)

- Staggered Coupling Approach, FEM and SBI share nodes on the virtual (infinite) boundary:

  1. Solve full time step within the FEM domain
  2. Set interface traction in SBI equal to the traction from FEM
  3. Solve Full time step within SBI.
  4. Set displacement of the shared nodes in FEM equal to the displacement in SBI.
  5. Return to Step 1 to advance to the next time step



Hybrid Method

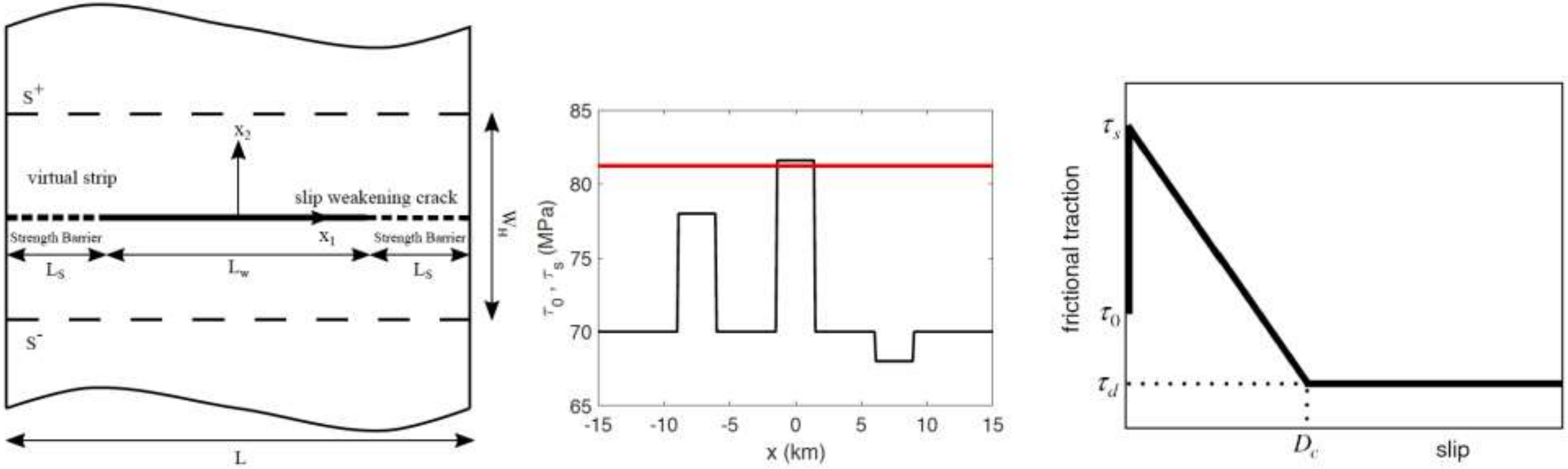
SCEC Benchmark TPV205 Verification

Problem Setup

- The Problem is a 2D in-plane fault, govern by the linear slip-weakening friction model.

$$\tau_f(\delta) = \begin{cases} \tau_s - (\tau_s - \tau_d)\delta/D_c, & \delta < D_c \\ \tau_d, & \delta \geq D_c \end{cases}$$

- The bulk is linear elastic homogenous under plane strain condition.

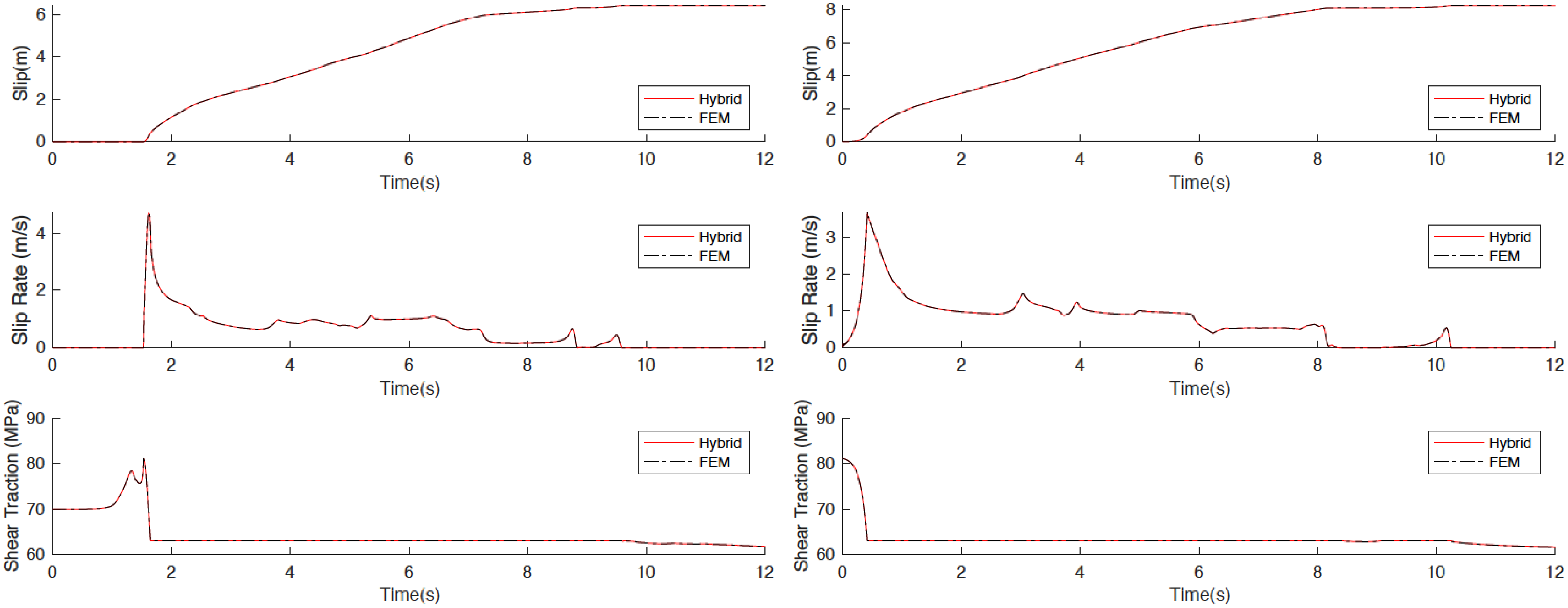


TPV205 Problem Description

Results

Time history comparison

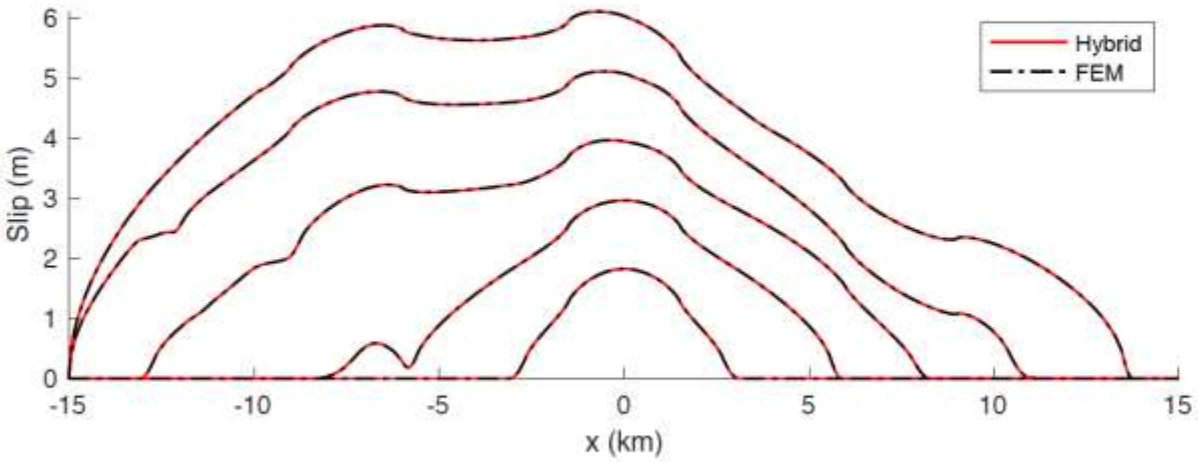
- Slip, Slip rate, Shear traction time history plots of points at the center of the fault and 4.5km away from the center of the fault

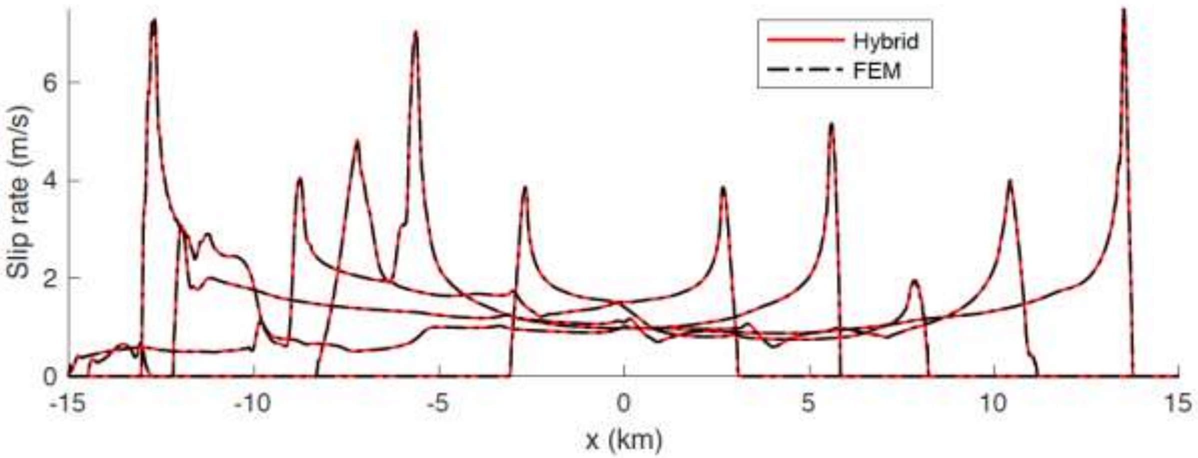


Results

Spatial comparison

- Snapshots of slip and slip rate at consecutive time steps.





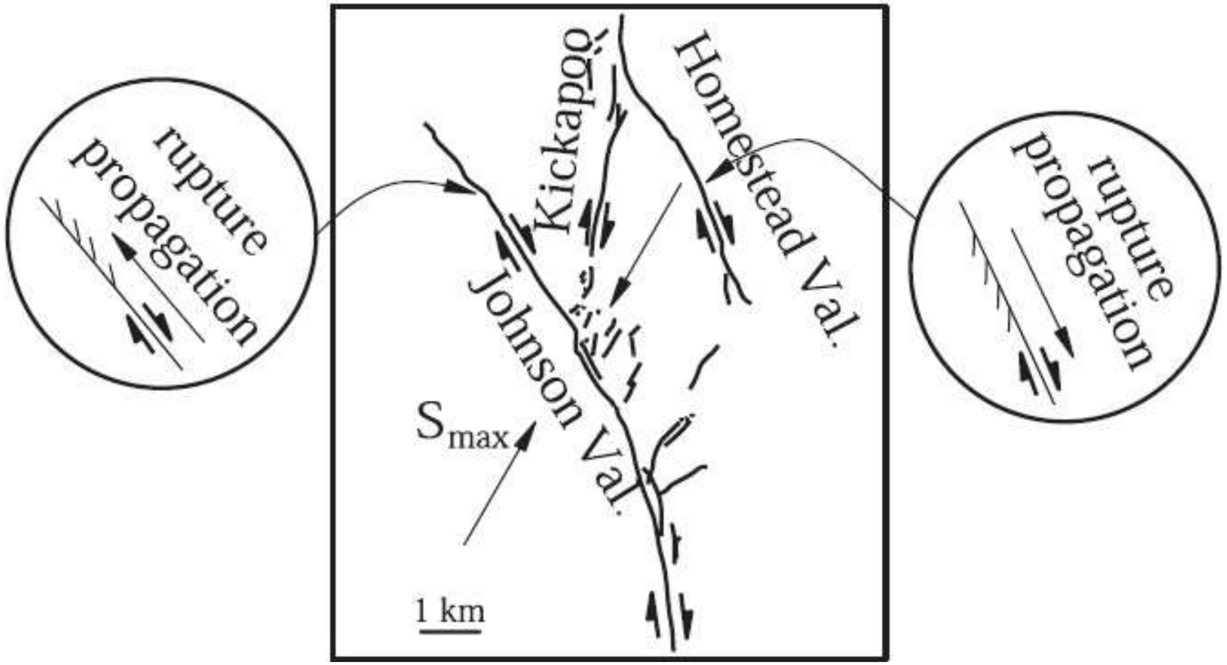
- The method has been verified extensively and was shown to be accurate, stable, and does not suffer from artificial reflections at the virtual boundaries

[Ma et al, IJANMG 2018 for details]

Effect of small scale branches on rupture dynamics

- Off-fault damage has been routinely modeled either using isotropic plasticity or scalar continuum damage theories due to computation limit.
- We hypothesize that small scale fractures may influence rupture characteristics as well as near source radiation fields in ways not captured by the continuum inelasticity approaches above.

Landers 1992



Johnson Valley 1992 rupture, at start of the Landers 1992 earthquake, bending along the Kickapoo (or Landers) fault zone [Poliakov et al. 2003]

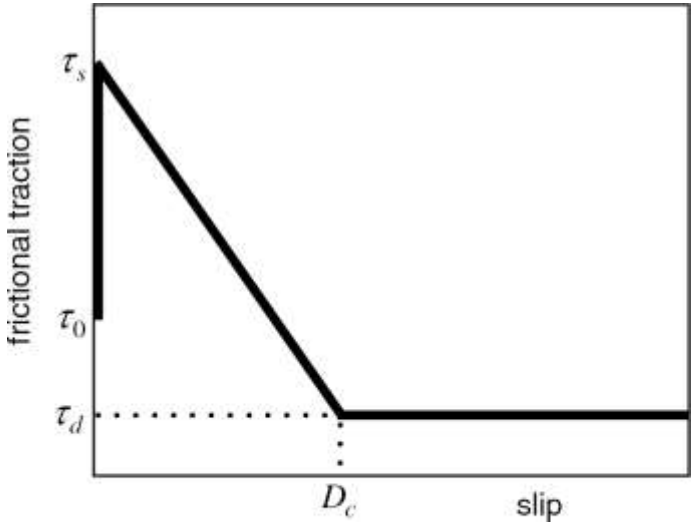
Ma, X. and Elbanna, A. E. "Dynamic Rupture Propagation on Fault Planes with Explicit Representation of Short Branches". *Earth and Planetary Science Letter,* 2019 (Under Review, available on EarthArXiv)

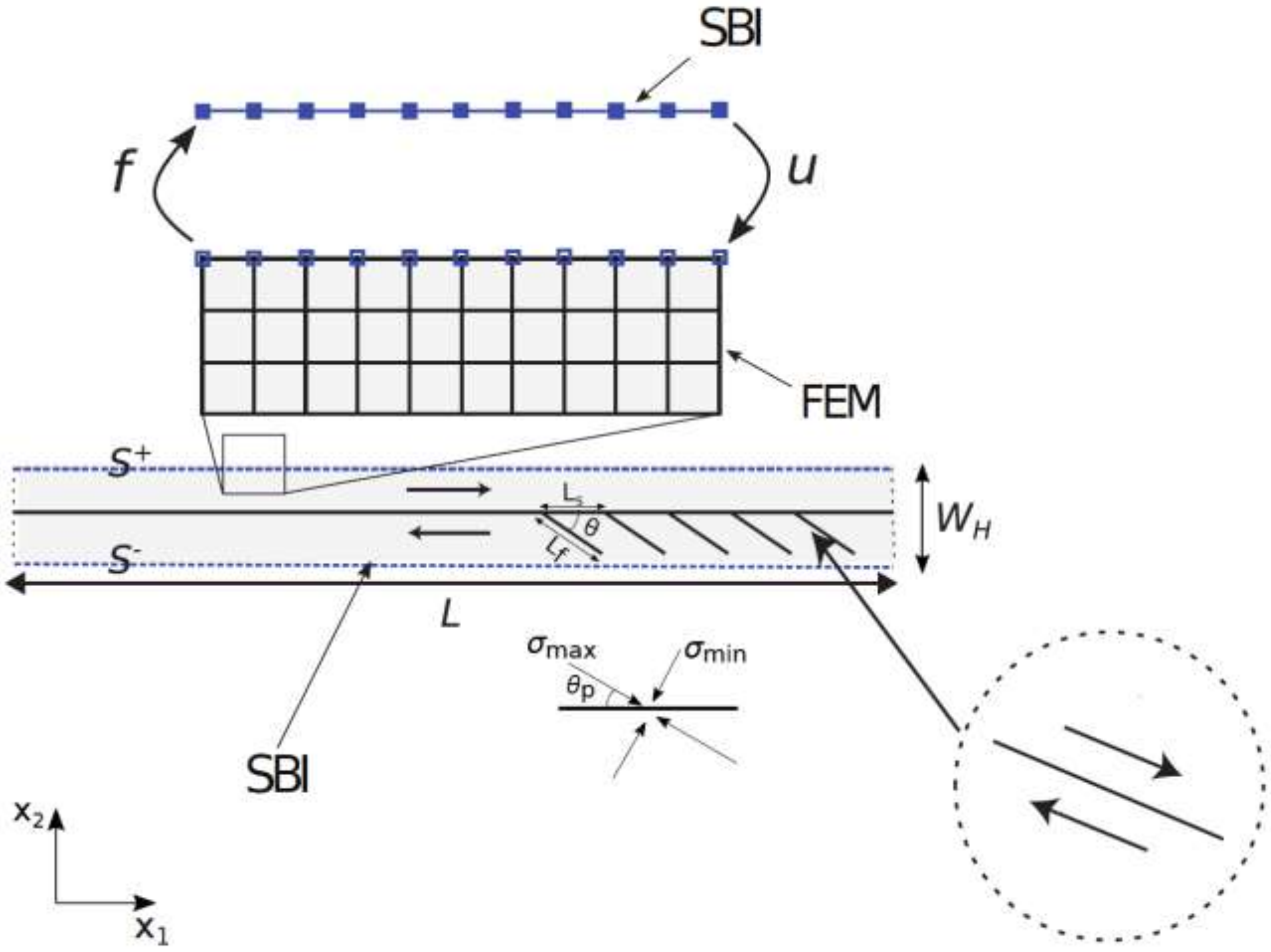
Effect of small scale branches on rupture dynamics

Model Setup

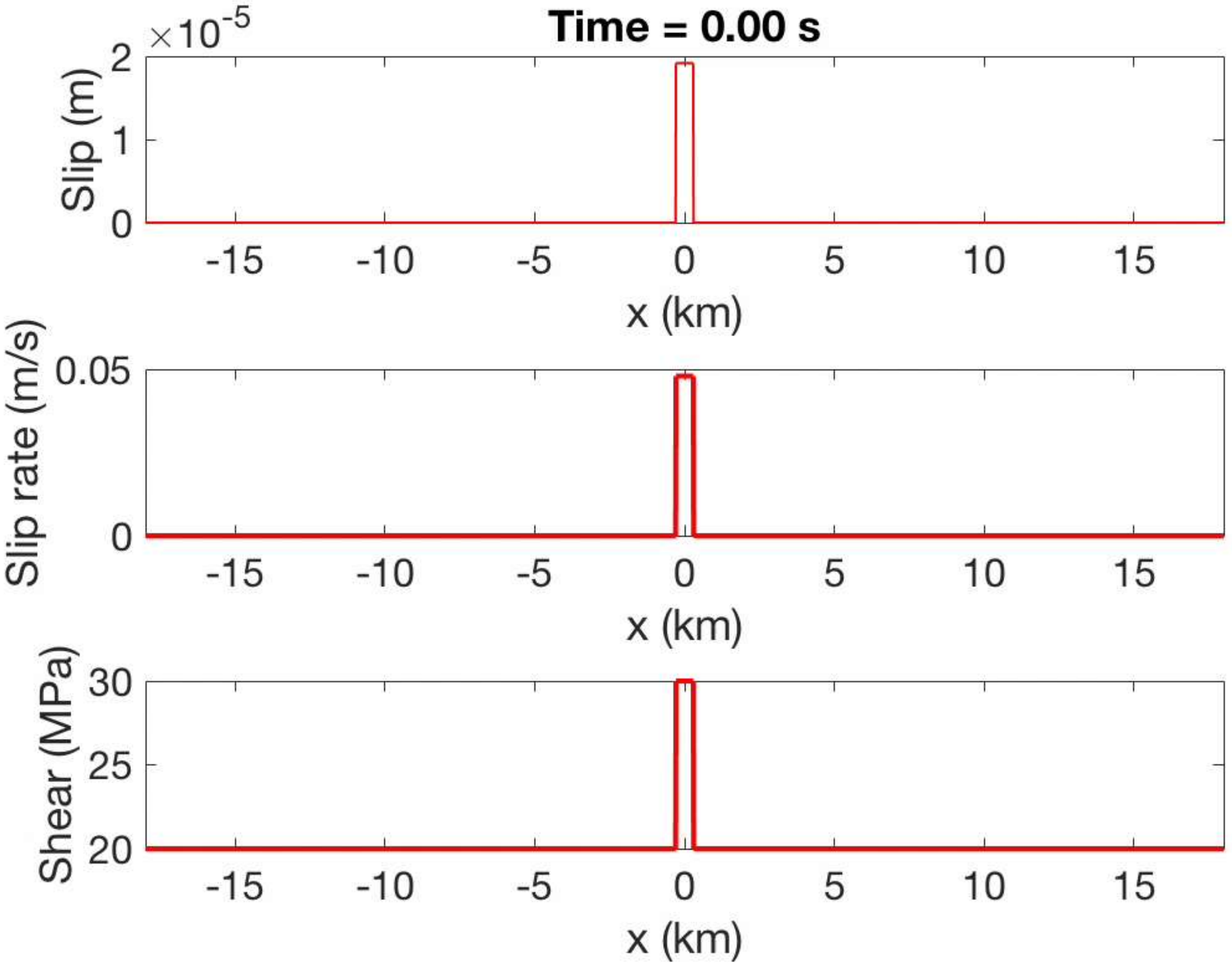
- A planar main fault is in a linear elastic medium under plane strain condition.
- The secondary fault branches are explicitly modeled
- The shear and normal stress along the faults are consistently resolve from the background stress.
- All fault are governed by the slip weakening friction model
- Length of the secondary fault is R , the spacing is R. The main fault length is 60R.

$$R = \frac{\mu D_c}{\tau_s - \tau_d}$$



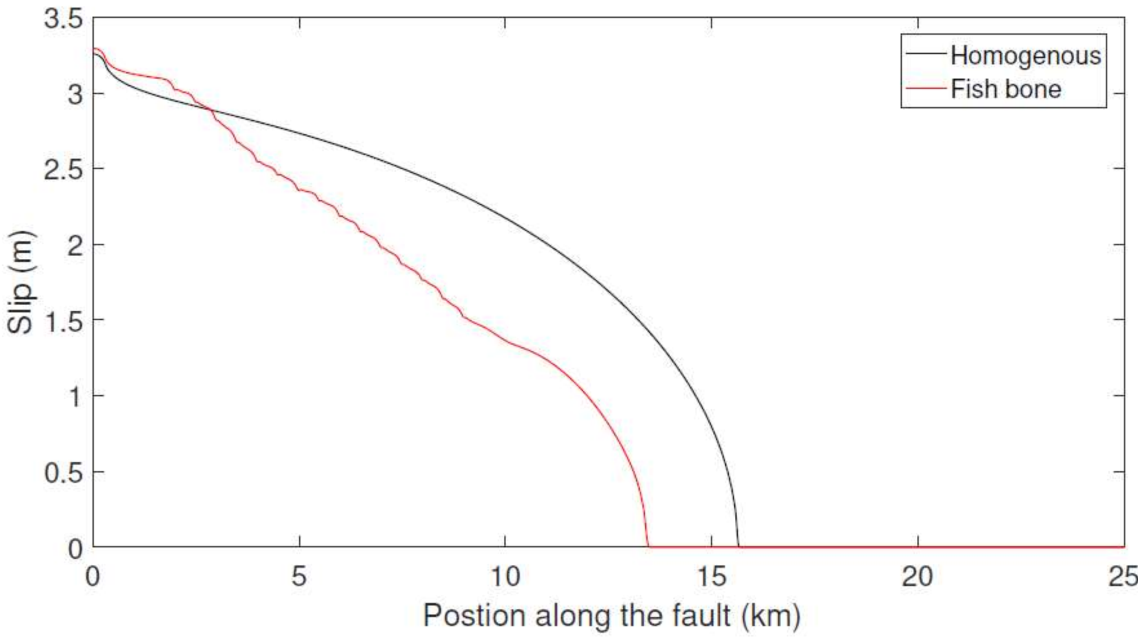


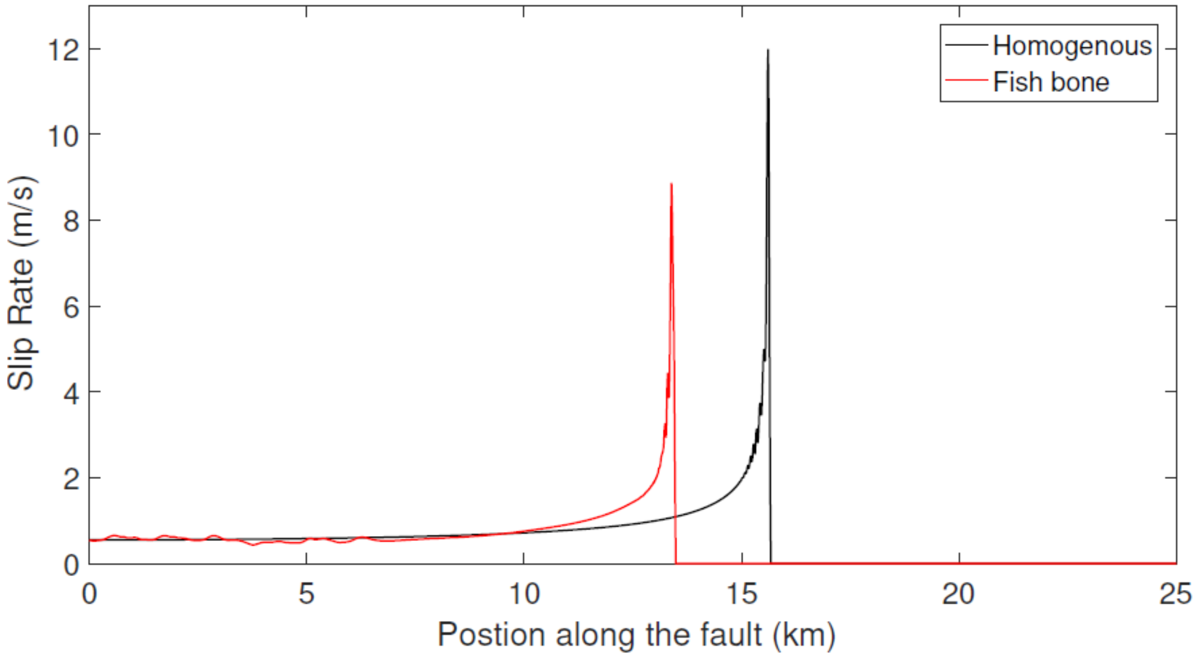
Results
Heterogeneity distribution after rupture events



Results
Slip, slip rate variation

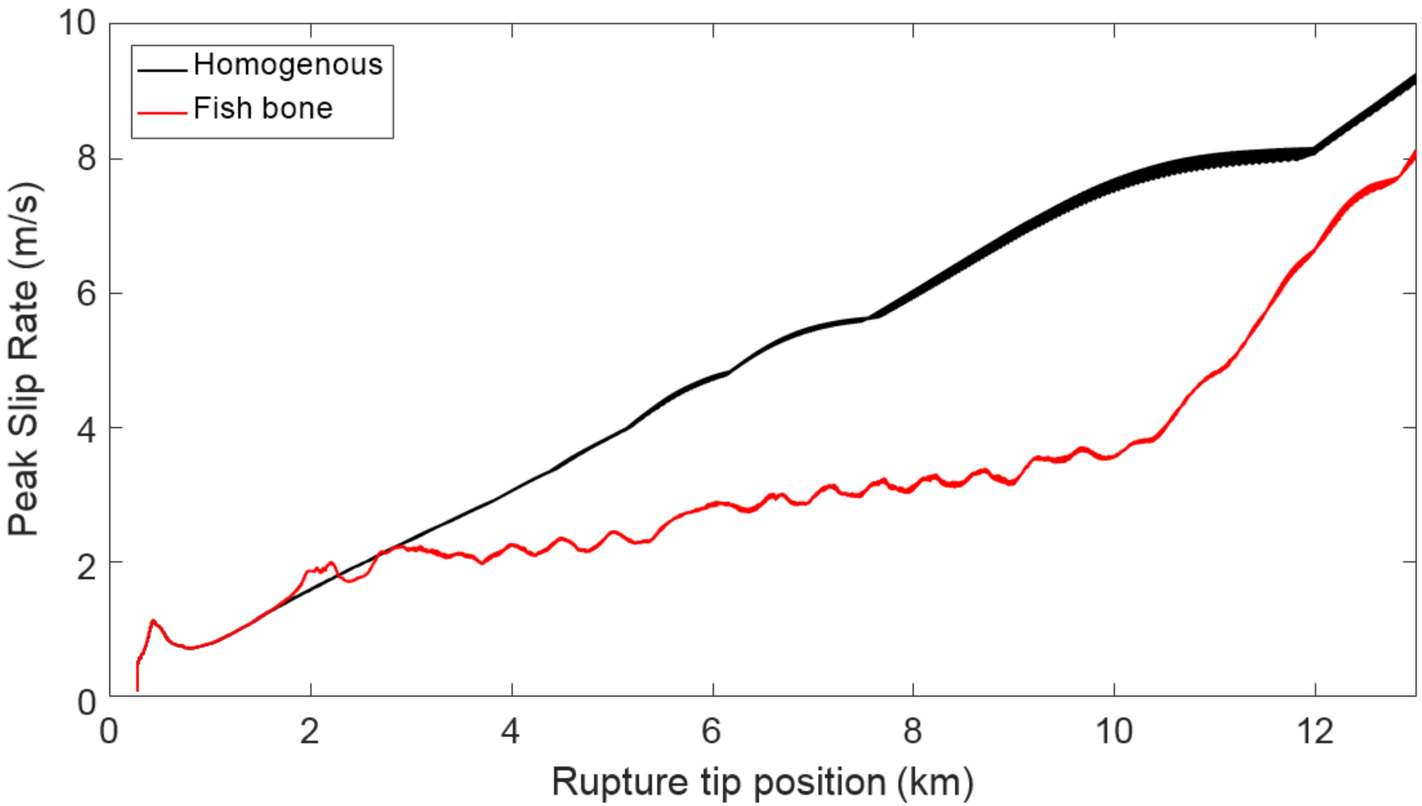
- The fish bone case accumulates less slip than the homogeneous case.
- The fish bone structure slows down the main fault rupture.





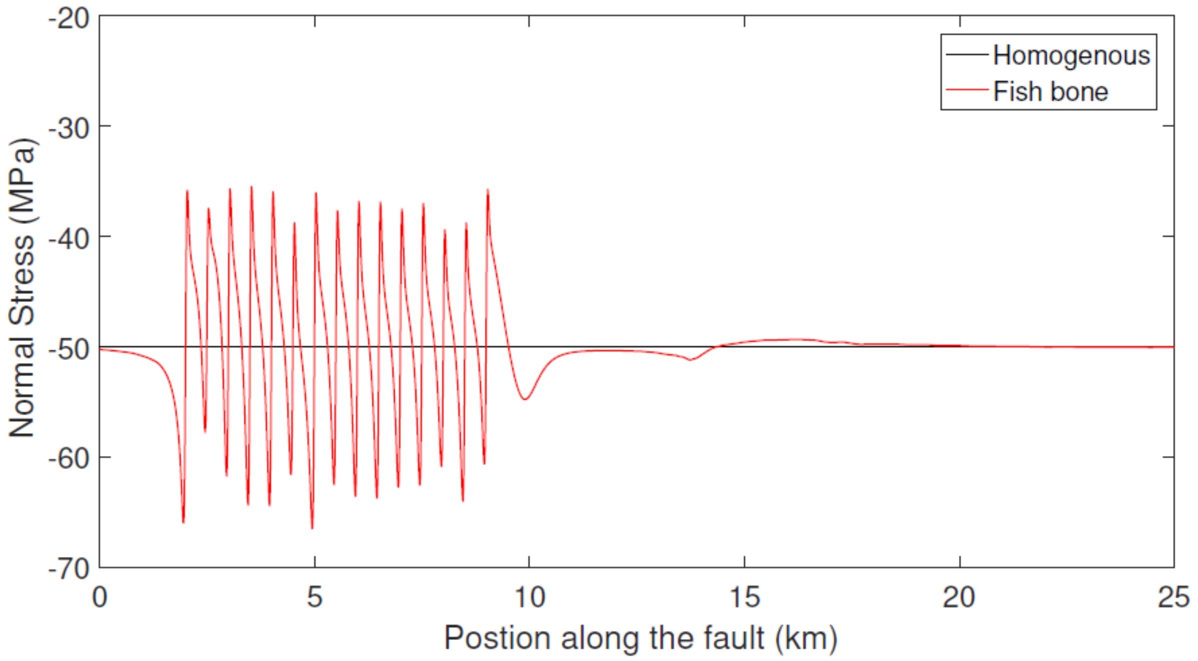
Results
Peak Slip rate variation

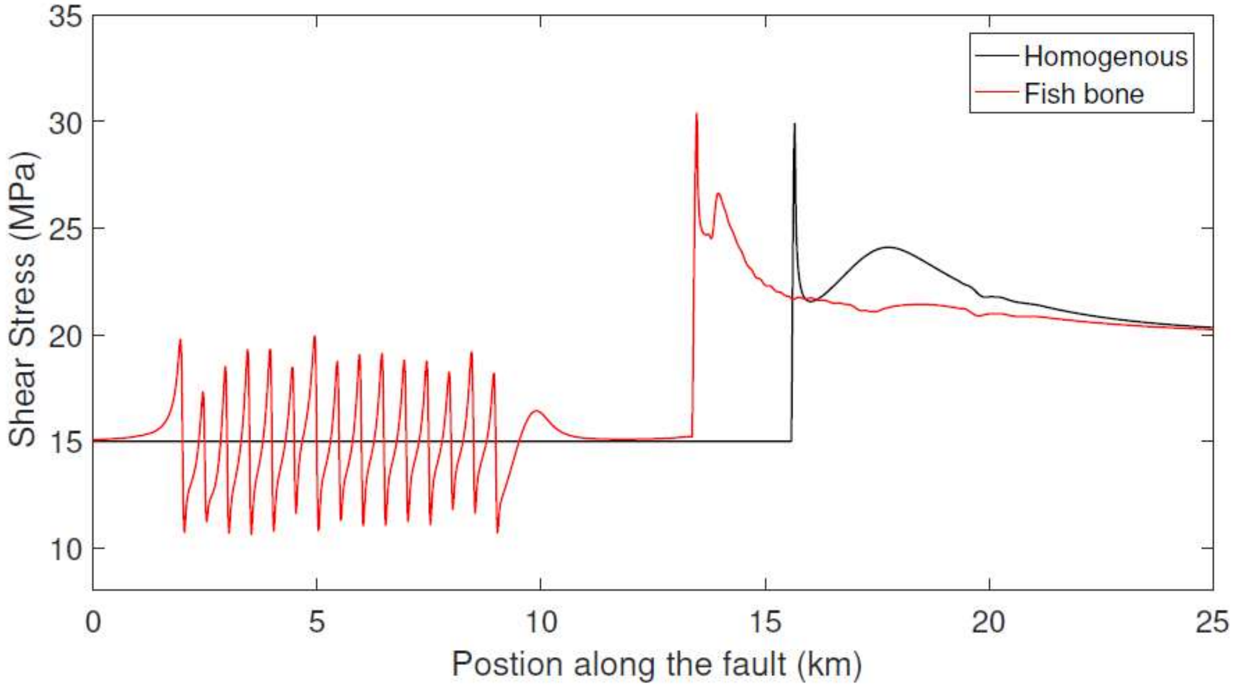
- The fish bone structure significantly reduce the peak slip rate.
- With fish bone structure, the peak slip rates shows increased high frequency oscillations.



Results
Stress heterogeneity on the fault

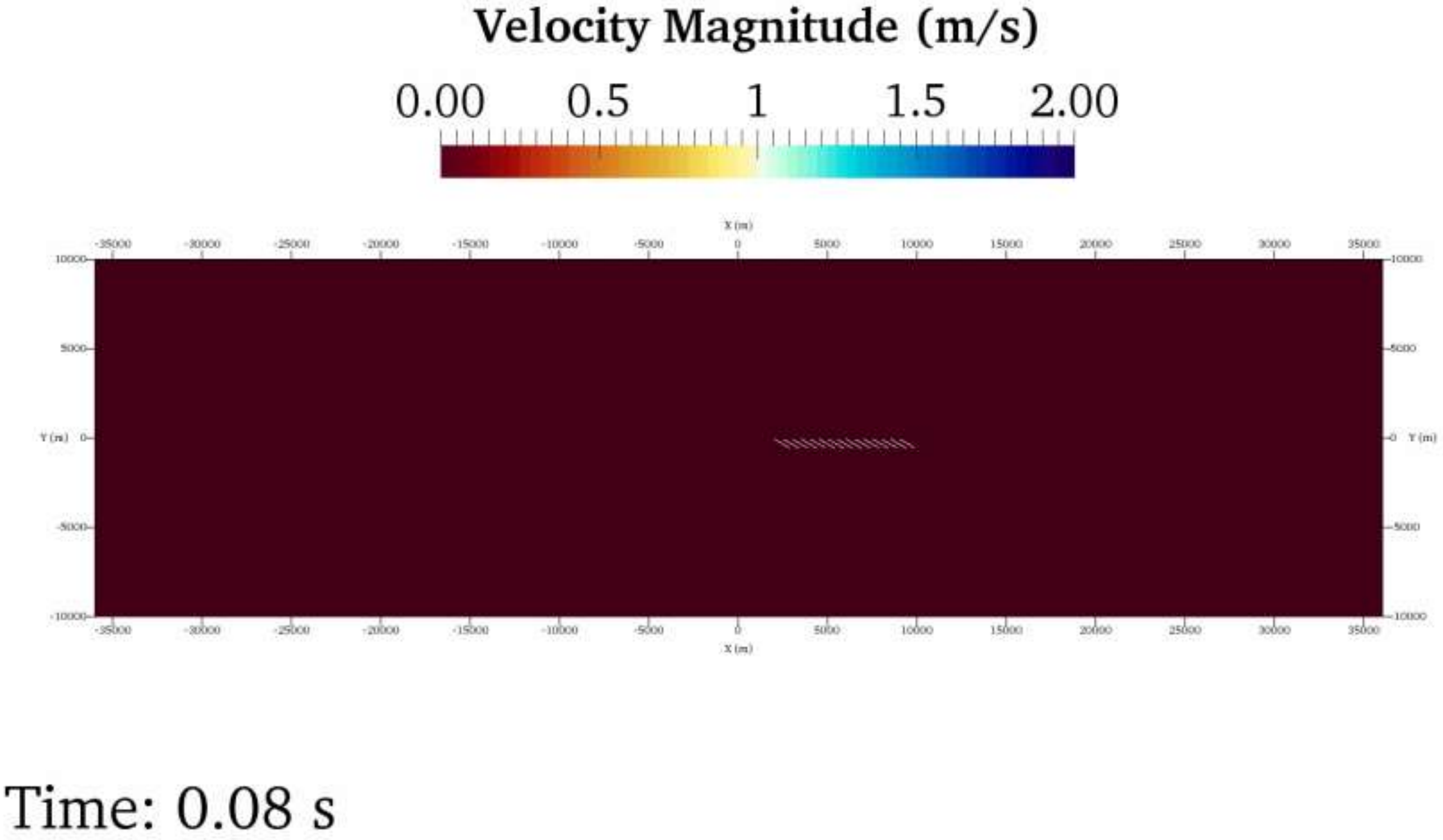
- Significant stress heterogeneity caused by the fish bone structure.
- The normal stress variation caused by the fish bone structure interaction has potential in promoting fault opening.
- These stress variation can not be captured if an isotropic plasticity or scalar continuum damage models are used





Results
High Frequency Generation

- Concentric fringes due to high frequency scattering are observed.
- These high frequency scatterings are emerging from the interference between seismic radiation from the main and secondary faults.



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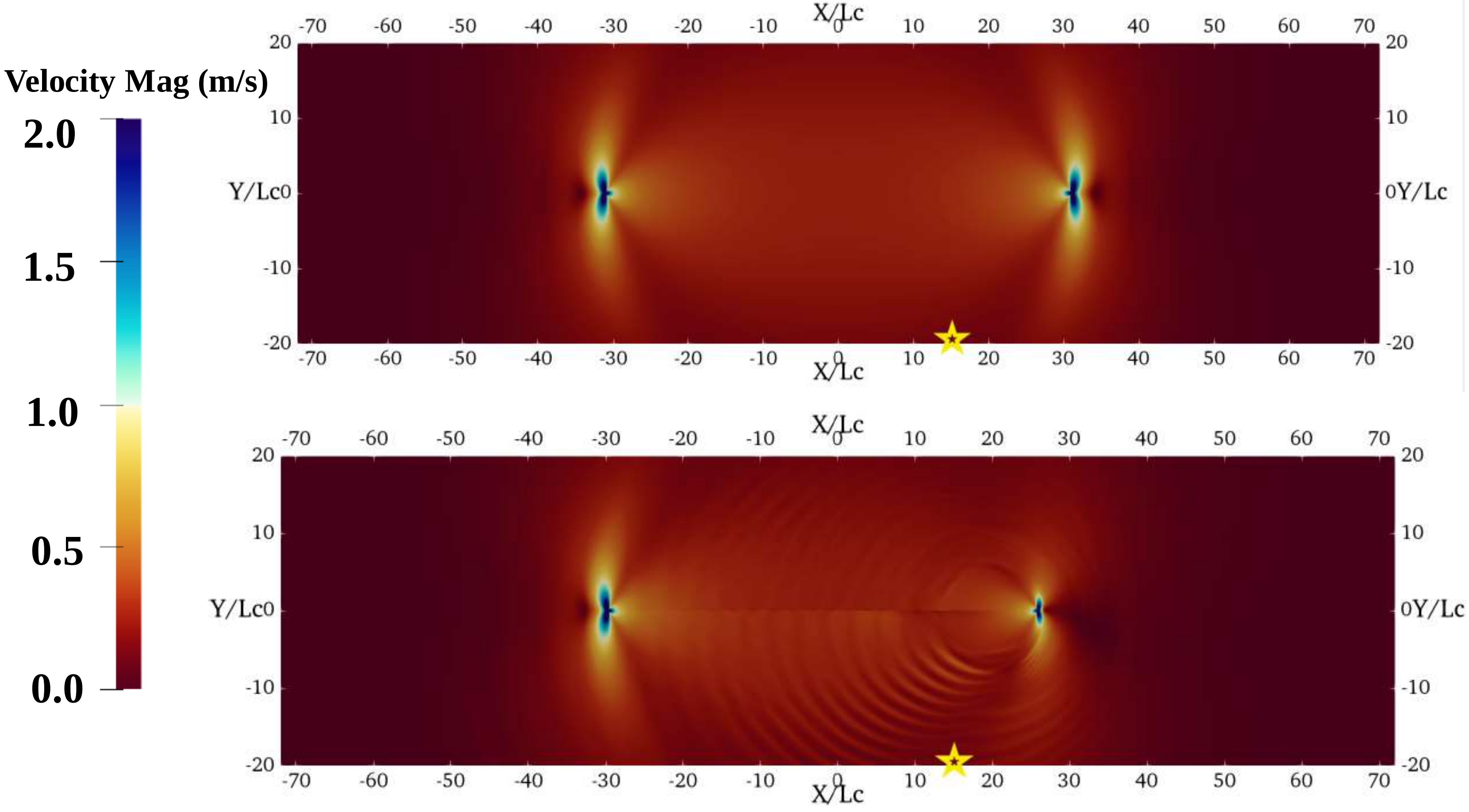
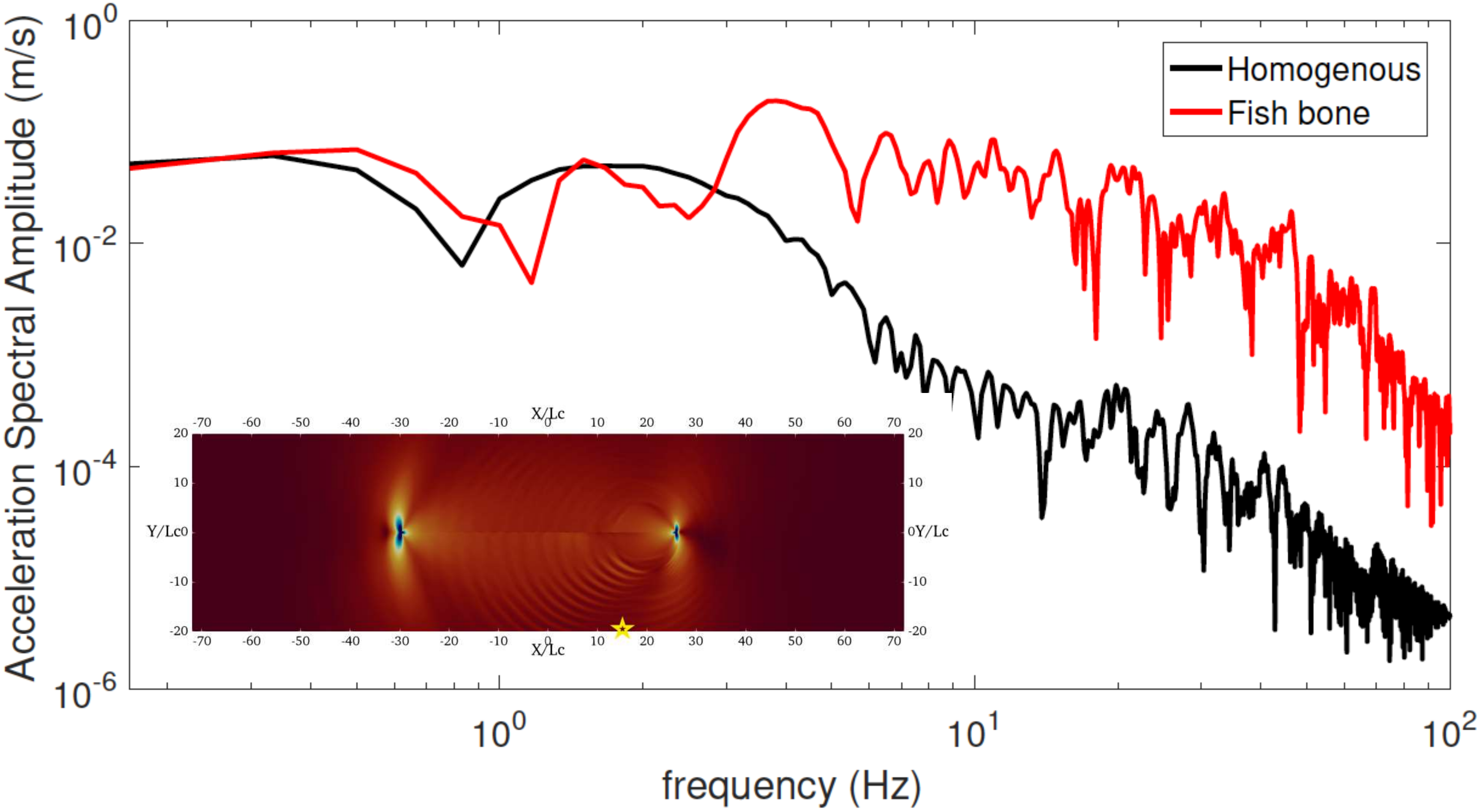


Results
High Frequency Generation

- Large amount of high frequency wave generated due to the existence of fish bone structure.
- Consistent with field observation (2-20 Hz)



Hybrid Method Earthquake Cycle simulation

- Earthquake Dynamic Rupture simulation. (Seismic Period order of Seconds)
  - Material nonlinearity
  - Geometrical nonlinearity
- Earthquake Cycle simulation (Seismic Period + Aseismic Period)

		Seismic Period		Aseismic Period
Earthquake Cycle Simulation	=	Explicit Dynamic (Time step bounded by CFL condition) Small Time step ~O(millisec)	+	Implicit Quasi-Dynamic (Adaptive Time Stepping) Large Time Step ~O(years)

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The Quasi-Dynamic Problem

Governing equations:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = \overset{0}{\cancel{\rho \ddot{u}}} \text{ in } \Omega$$

$$\sigma_{ij} n_j = \tau_{SBI} \text{ on } S_{SBI}$$

$$u_i = u_i^0 \text{ on } S_u$$

$$R_{ki}\,(u_i^+ - u_i^-) = \delta_k \text{ on } S_f$$

Rate and state friction model:

$$f(V, \theta) = a \sinh^{-1}\left[\frac{V}{2V_o} \exp\left(\frac{f_o + b \ln\left(\frac{\theta V_o}{L}\right)}{a}\right)\right]$$

$$\frac{d\theta}{dt} = 1 - \frac{\theta V}{L}$$

Regularization through radiation damping (Rice 1993):

$$T^f = F(V, \theta) + \eta V, \qquad \eta = \frac{\mu}{2 c_s}$$

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The Quasi-Dynamic Hybrid Scheme

Fault traction as a Lagrange Multiplier

$$\mathbf{K}u(t) + \mathbf{L}^T\left(\tau^{SBI}(t) + T^f(t)\right) = \mathbf{F}(t)$$

$$\mathbf{L}u(t) = \mathbf{D}(t)$$

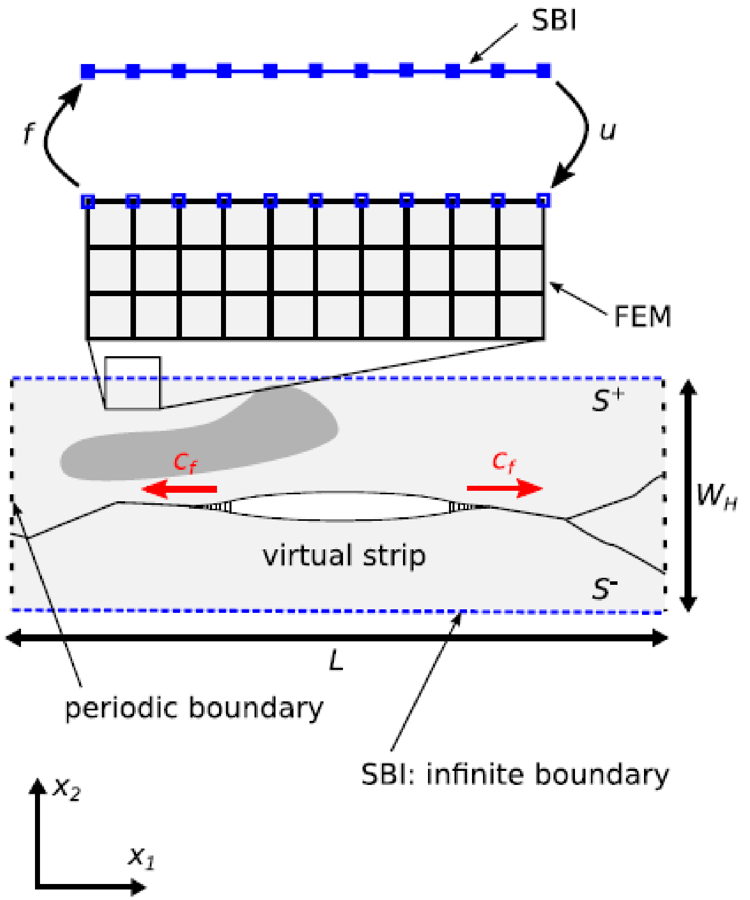
Time dependent loading

τ

Traction on the FEM virtual boundaries

Fault slip

- Predictor-Corrector approach to estimate $\tau^{SBI}(t)$. Start with: $\tau(t) = \tau(u(t - \Delta t))$
- Compute T^f and solve the friction equation to calculate slip rate V and time march to $t + \Delta t$.
- Adaptive time marching scheme using RK45 to integrate for fault slip and state variable.



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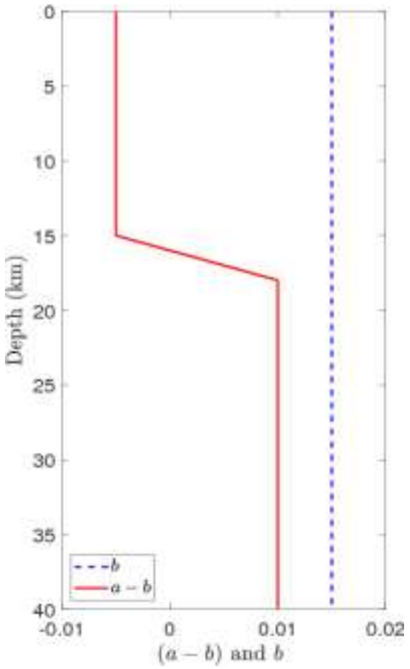
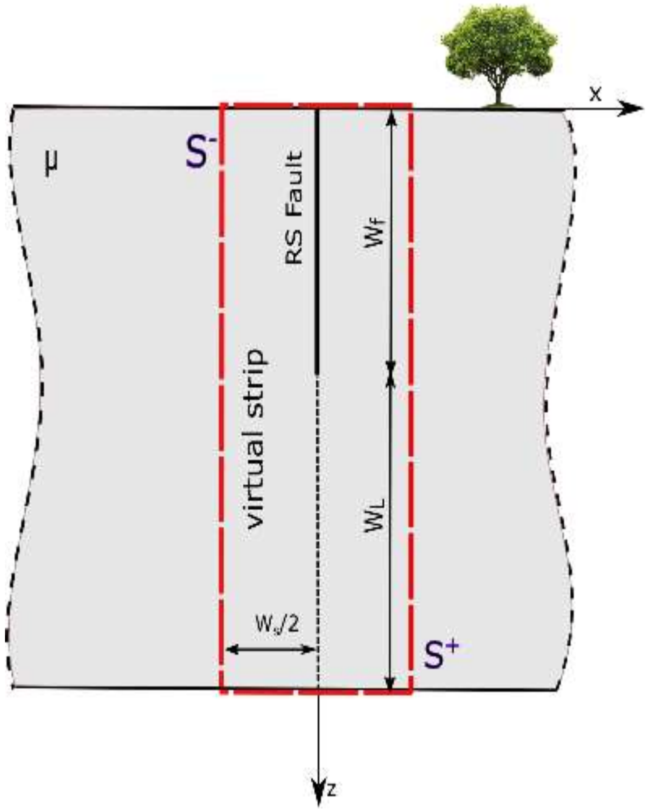
Earthquake Cycles: SCEC-SEAS benchmark Verification

- SCEC SEAS benchmark BP-1.
- 2D Anti-plane setup.
- Virtual strip width (arbitrary) $W_s = 1$ km.
- Fault width $W_f = 24$ km





Benchmark Problem Setup

More details and a complete list of parameters available at:
http://scecdata.usc.edu/cvws/seas/benchmark_descriptions.html

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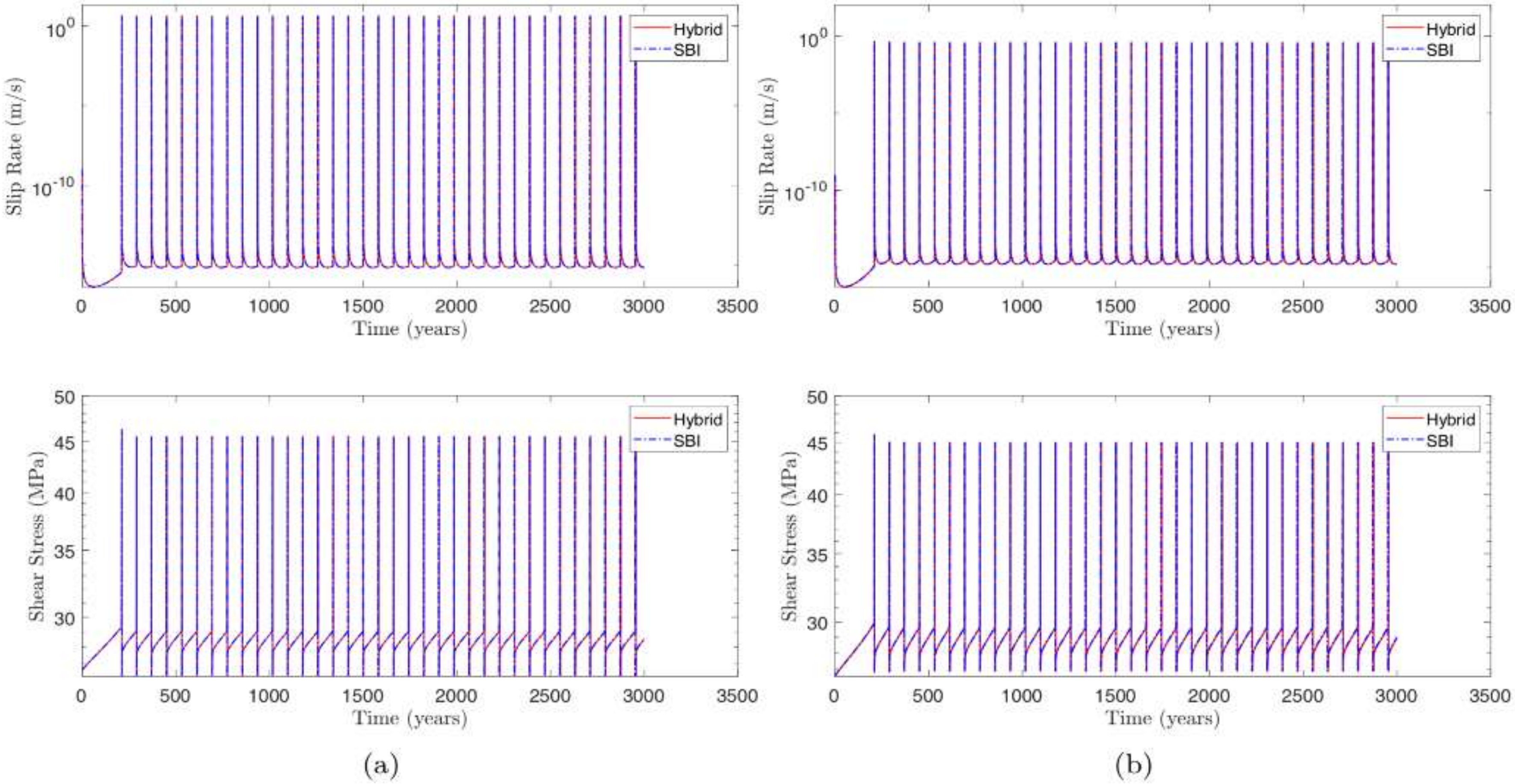
SEAS BP-1 Verification

- Slip rate and shear stress at two different stations along fault, (a) 0 km and (b) 7.5 km.



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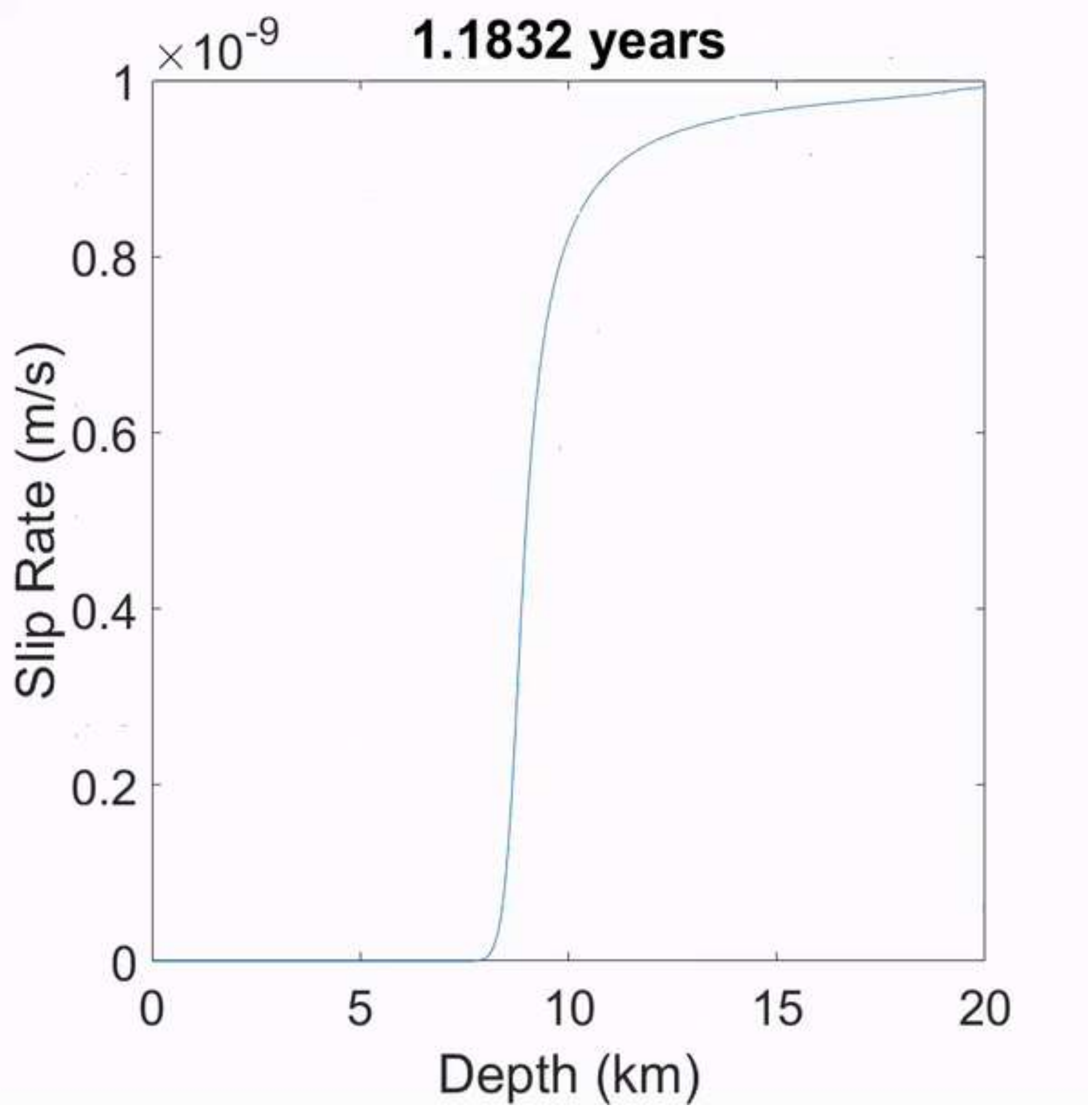
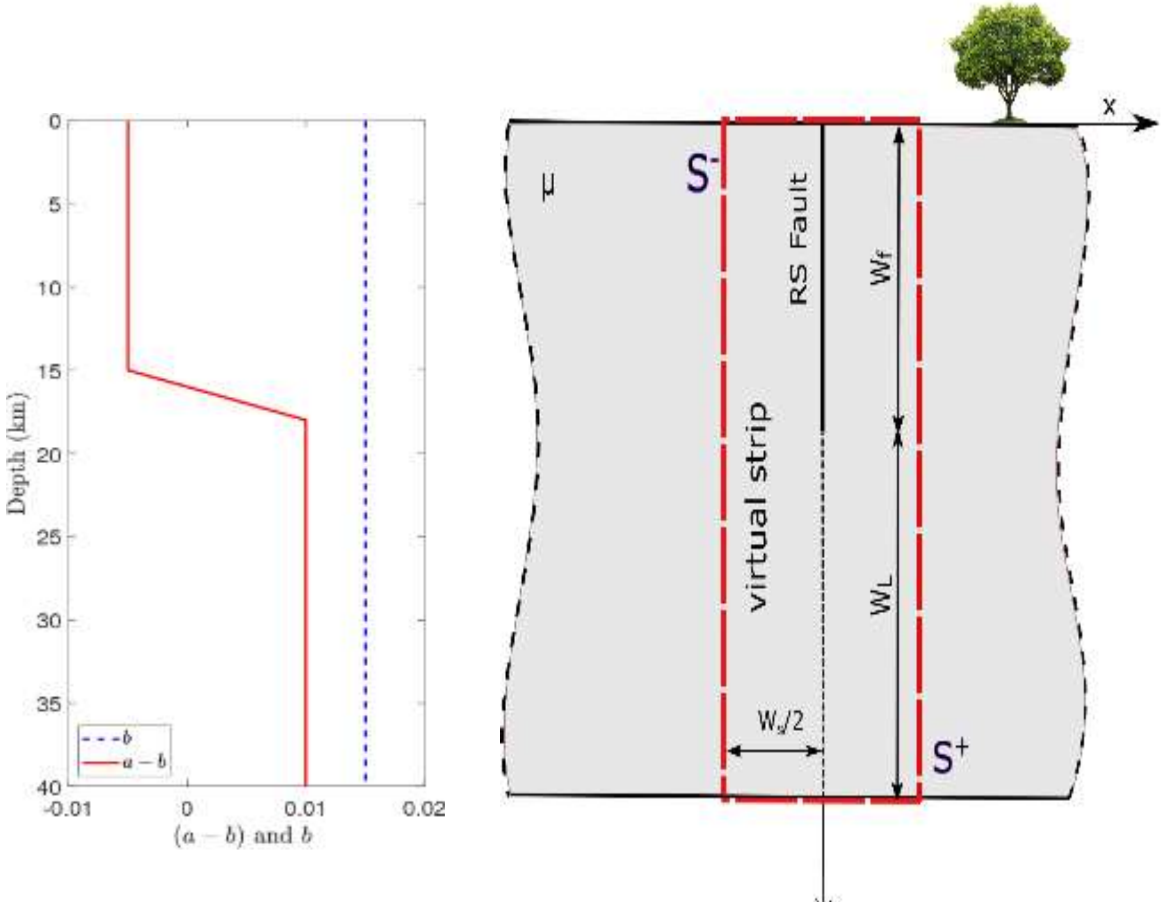
Towards Cycle Simulations with Inertia

- Steps to incorporate inertia in the current framework:

1. Retain the quasi-dynamic framework with adaptive time-stepping for inter-seismic periods to model spontaneous rupture nucleation [Abdelmeguid et al, earthArXiv, 2019].
2. Utilize the hybrid scheme developed for dynamic rupture that integrates explicitly in time for earthquake event [Hajarolasvadi and Elbanna, GJI, 2017; Ma et al, IJNAMG, 2018].
3. Implement absorbing boundary conditions to ensure waves escape the domain.
4. Switch between dynamic and quasi-dynamic based on the ratio of inertia term to other terms in the governing equation within the region of interest. Switching from QD to Dynamic is based on a velocity threshold.

Test Problem setup: Similar to BP-1

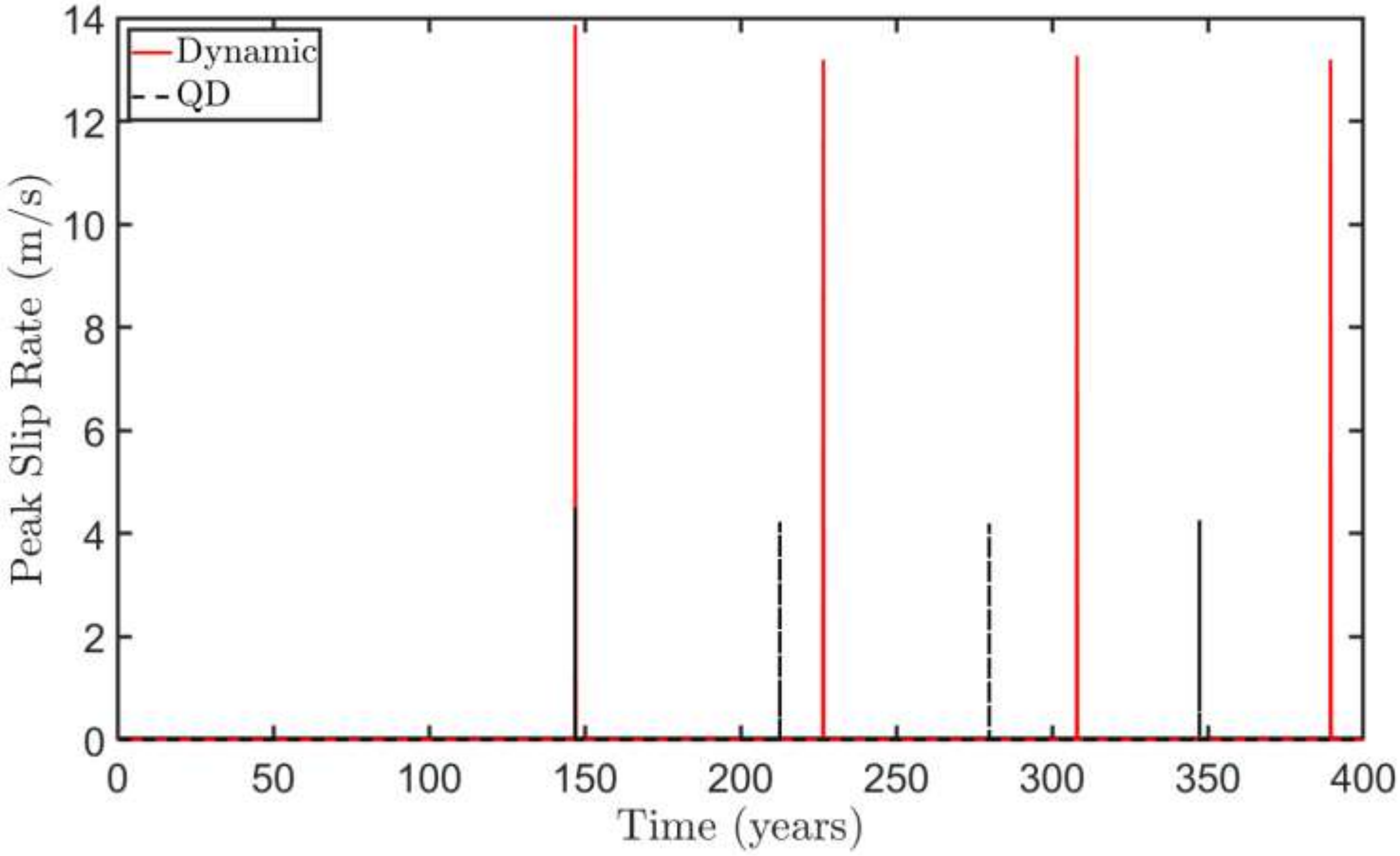




Setup for the full cycle simulation: 2D anti-plane rate and state fault in a homogeneous medium

Results: Compare with Q-D Solution

- Dynamic simulations give higher slip rates and larger slip per event.
- Dynamic events have a larger recurrence periods.



Conclusions

- The hybrid scheme provides a pathway for modeling complex fault zones with high resolution.
- Explicit representation of small-scale branches reveal some unique phenomena that would have been smeared out using continuum inelasticity. These phenomena include, for example, strong stress heterogeneities on the main fault, enhanced high frequency generation, and supershear transition on the main fault even if the conditions are unfavorable.
- Extending these simulations to the context of earthquake cycles will enable testing which observations remain robust and independent of the particular initial conditions considered in single rupture simulations. Initial progress reported in *Abdelmeguid et al*, *2019 EarthArXiv* with additional developments related to incorporation of inertia was presented here. Further extensions related to other forms of fault zone complexities are also planned.
- Small scale physics and geometry matter, perhaps even more with inertia present!

Acknowledgements

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SCEC

an NSF+USGS center

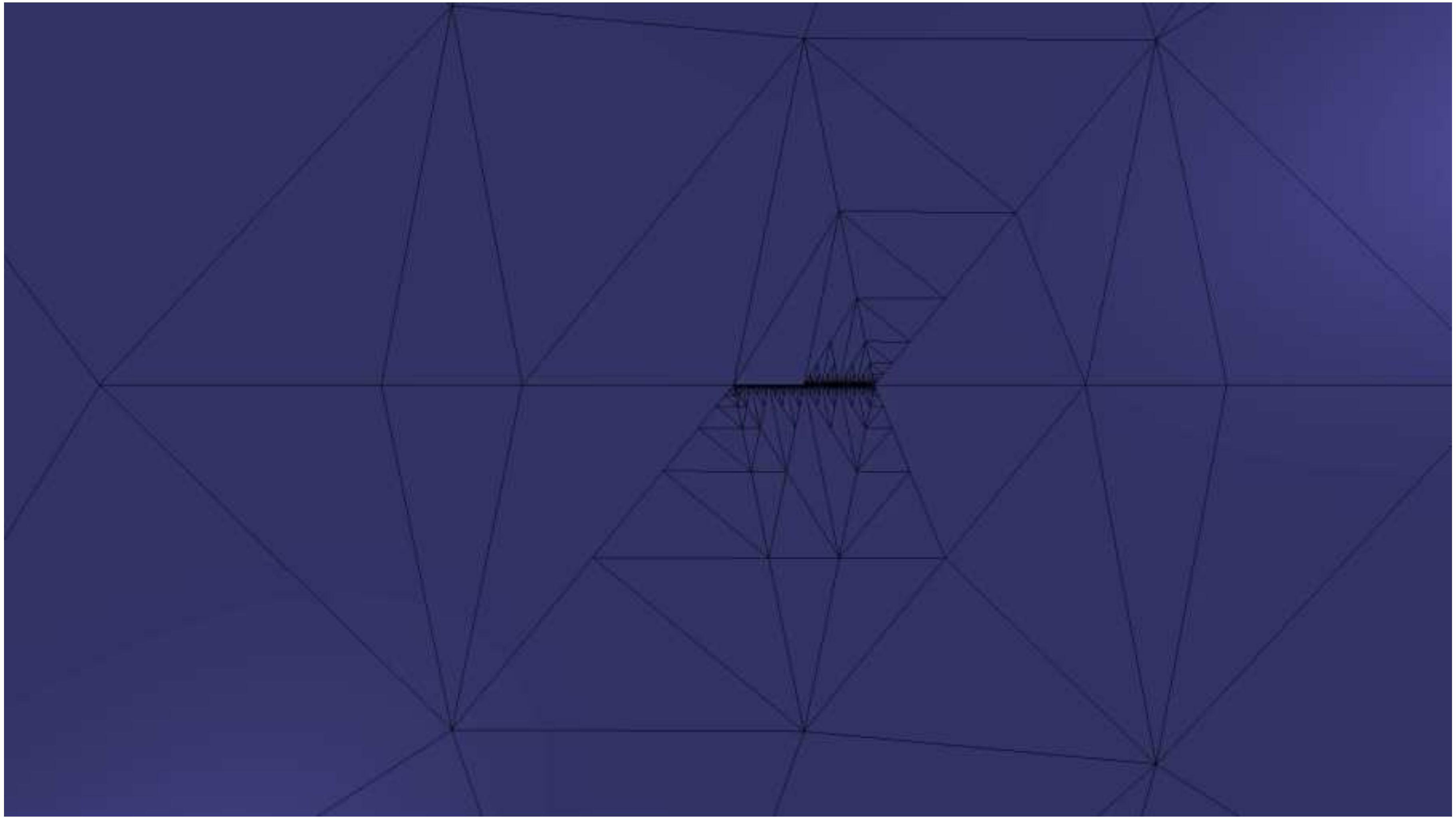
GSCO2

Center for Geologic Storage of CO_2

U.S. DEPARTMENT OF ENERGY

Office of Science

Asynchronous Space-Time Discontinuous Galerkin Method SCEC-TPV205



- Adaptivity in Space and Time
- 3D X Time element was developed.

Collaboration work with Prof Haber at UIUC

References

- Ma, X., Hajarolasvadi, S., Albertini, G., Kammer, D., Elbanna, A. “A hybrid finite element-spectral boundary integral approach: Applications to dynamic rupture modeling in unbounded domains”, *International Journal for Numerical and Analytical Methods in Geomechanics*, 2018.

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