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ABSORPTION CORRECTION FOR COMPUTATIONS OF A SEISMIC GROUND RESPONSE

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In computations of synthetic seismograms for purposes of ground-motion studies, the absorption should be always taken into account. Otherwise, serious errors in the seismic hazard estimation can be made. Three main approaches for including absorption into synthetic seismograms are: (i) by complex velocities in the frequency domain, (ii) through stress-strain relations and equations of motion in the time domain, or (iii) by dissipation operators applied to the perfectly elastic solutions in the time or frequency domain. For the latter method, the wave field must be decomposed into elementary waves of known travel times.

In Zahradník *et al.* (1990), an alternative method (iv) has been suggested recently, i.e., the approximate absorption correction similar to (iii), but applicable to the wave fields not decomposed into elementary waves. A need for such an approach is quite evident. For example, if the finite-difference method is used for generating a suite of synthetic ground motions with various absorption models, the (expensive) finite-difference code may be executed only once, without absorption, and then the approach (iv) may be repeatedly used. This idea has been put forward before (e.g., Boore *et al.*, 1971; Zahradník and Hron, 1987; Vidale and Helmberger, 1988). The intention of this note is to clarify some computational aspects of that simple technique (iv) and to prove its efficiency for ground-response analyses.

APPROXIMATE ABSORPTION CORRECTIONS

Consider the *SH* wavefield and the absorbing medium. Let the medium be heterogeneous, i.e., the phase velocity $c(f)$ is dependent on spatial coordinates, but restricted to spatially constant $Q(f)$. Assume, for now, a single elementary wave, whose waveform at $\tau = 0$ is $g(t)$. Here τ denotes the travel time $\tau = \int ds/c(f_r)$ along a ray. Due to absorption the waveform at $\tau \neq 0$ becomes

$$p(t) = D(t; \tau) * g(t - \tau), \tag{1}$$

where $D(t; \tau)$ is the dissipation operator given by equations (34) through (37) of Müller (1983). Now consider a medium in which, instead of the single wave, the nonabsorbing response consists of N superimposed elementary waves

$$r(t) = \sum_{j=1}^{N} g_j(t - \tau_j). \tag{2}$$

Then the absorbing response is

$$p(t) = \int_{+\infty}^{-\infty} \sum_{j=1}^{N} D(t - t'; \tau_j) g_j(t' - \tau_j)\, dt', \tag{3}$$

not explicitly expressed via the nonabsorbing response $r(t)$. The application of equation (3) requires a knowledge of all g_j and τ_j, i.e., the wave field composition must be known. If, however, the parameter τ_j of the operator D in equation (3) is

substituted by a function of t', independent of j, e.g., $t' - T_m$, we may write

$$p(t) = \int_{+\infty}^{-\infty} D(t - t'; t' - T_m) r(t')\, dt'. \tag{4}$$

Hence we get an approximate absorption correction applicable *a posteriori* to the perfectly elastic response $r(t)$ not decomposed into elementary waves, as desired. For reasons explained in Zahradník *et al.* (1990) we use in (4) the parameter $t' - T_m$, where T_m denotes the time at which $g_j(t)$ takes its maximum. If this time is different for the individual g_j, the T_m is averaged from all of them. The shorter is the duration of the excitation (hence also that of g_j), the better is the approximation (4), being quite exact for the impulse excitation. Realistic earthquake excitations can be convolved with the impulse response.

Consider the absorbing medium described by the causal power-law Q model of Müller (1983):

$$Q(f) = Q(f_r)\left(\frac{f}{f_r}\right)^{\gamma}. \tag{5}$$

Here f_r is a reference frequency, and γ characterizes a particular frequency dependence of the quality factor Q.

For $\gamma = 1$ (linear Q) the approximate absorption correction of equation (4) gives a simple exponential windowing of the nonabsorbing response $r(t)$

$$p(t) \simeq \exp\left\{-\frac{\pi(t - T_m)}{q}\right\} r(t), \tag{6}$$

where $q = Q(f_r)/f_r$. For $\gamma = 0$ (constant Q) we get

$$p(t) \simeq F^{-1}(P(f)), \tag{7a}$$

$$P(f) \simeq \int r(t') \exp\left\{-\frac{\pi f(t' - T_m)}{Q}\left[1 - \frac{2i}{\pi}\ln\left(\frac{f}{f_r}\right)\right]\right\} \times \exp\{-2\pi i f t'\}\, dt', \tag{7b}$$

where F^{-1} denotes the inverse Fourier transform. Some attention should be paid to an efficient numerical evaluation of $P(f)$ (see the Appendix).

While our approach is based on equation (4) in the time-domain, an alternative frequency-domain approach can be also used (D. C. Witte, written comm.). Since the multiplication in equation (4) transforms into a frequency-domain convolution, and since the dissipation operator D is slowly varying with time, the convolution can be restricted to a narrow frequency band. Hence, a good numerical efficiency can be expected, too.

A Canonical Basin Problem

Here we verify the approximate absorption corrections. We compare them against methods in which complete interference *SH* wave fields are computed by the finite-difference method with absorption exactly included in the time-domain equation of

motion. At the same time we address an important practical problem: how do different absorption corrections affect synthetic wave fields in studies of near-surface geological effects upon earthquake ground motions?

We use the well known test example of the sedimentary basin, first introduced by Boore *et al.* (1971) and presented in Figure 1 (see also Moczo *et al.*, 1987, and the references therein). The shear-wave velocity takes constant values inside the basin (β_s) and in the underlying rock (β_r): $\beta_s = 700$ m/sec, $\beta_r = 3500$ m/sec. The quality factor $Q(f)$ is constant throughout the whole model. Two absorption models are considered: the linear-Q model, $Q = 36.6\,f$ (i.e., $Q(f_r) = 20$, $f_r = 0.54$ Hz, where f_r is the predominant frequency of the excitation signal), and the constant-Q model $Q = 20$. The basin is excited by a plane SH wave, incident vertically from below. At the initial time $\tau = 0$, the incident wavefront is 100 m below the deepest point of the basin. The excitation signal is the Ricker wavelet of duration $T = 4$ sec; hence $T_m = 2$ sec. Five solutions for this model, corresponding to the six receivers of Figure 1, are compared in Figure 2.

The *approximate* solution for the *constant-Q* model is computed in two steps. In the first step the perfectly elastic response $r(t)$ is found by the finite-difference method. The second step consists of applying equations (7a) and (7b). We call this solution absorption correction 1 and denote it AC1.

The *reference* solution for the *constant-Q* model is computed by a time-domain technique based on Emmerich and Korn (1987) and Moczo (1989). The absorption is described by a partial differential equation coupled with a system of ordinary differential equations. These equations are solved by the finite-difference method. The denotation is EK.

The *approximate* solution for the *linear-Q* model is again computed in two steps. First, the perfectly elastic response $r(t)$ is found by the finite-difference method. Second, the approximate absorption correction (6) is applied to the response $r(t)$. This solution is called absorption correction 2 and denoted AC2.

The *reference* solution for the *linear-Q* model is provided by the finite-difference method as a whole. In this case the absorption is simply and quite accurately included in the time-domain equation of motion through an additional term $2\pi\rho(f_r/Q(f_r))\partial u/\partial t$. Hence the denotation AT.

The perfectly elastic response $r(t)$ is included in Figure 2 as the trace denoted ELASTIC.

As evident from Figure 2, the agreement between the exact and approximate absorption corrections is very good. In particular, the agreement for the

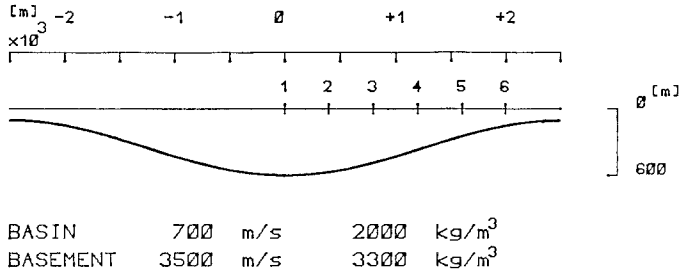

FIG. 1. Model of sedimentary basin used for checking the absorption corrections of the present paper by the finite-difference method in Figure 2. The receivers are denoted by numbers 1 to 6. The shear-wave velocities and densities are also shown.

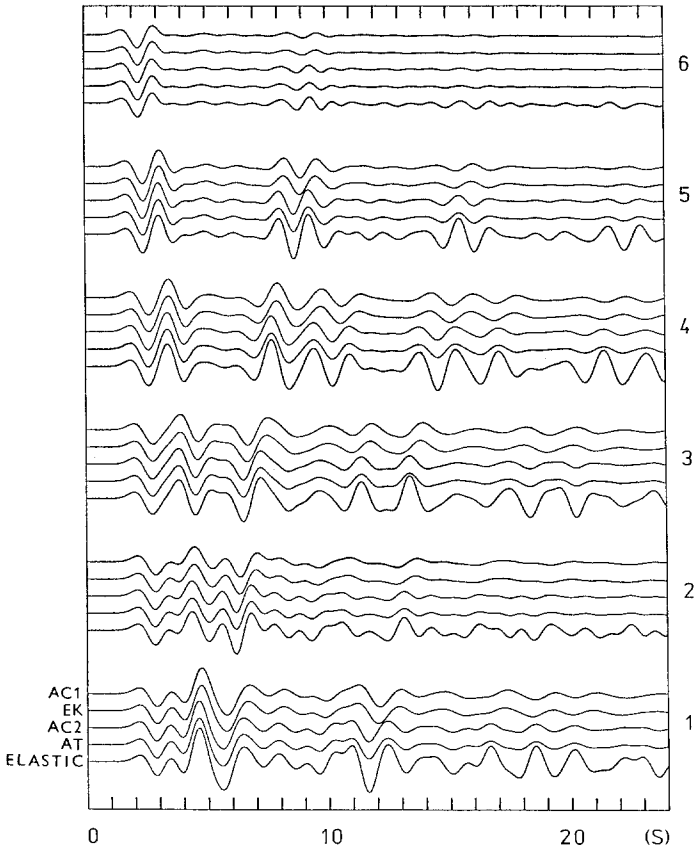

FIG. 2. A comparison of the absorbing-medium response $p(t)$ computed by four independent methods for six receivers of the basin model in Figure 1. EK and AT: the *reference* solutions for $\gamma = 0$ and $\gamma = 1$, respectively; AC1 and AC2: the *approximate* solutions of this paper for $\gamma = 0$ and $\gamma = 1$, respectively. ELASTIC denotes the perfectly elastic response $r(t)$.

constant-Q model, complicated by dispersion, is as good as that for the linear-Q model, where the dispersion is absent.

Naturally, the constant-Q and the linear-Q model provide different results. However, in many seismic-response analyses of near-surface geological structures, where the interest is in gross features of ground motions and not in details (mostly because of the data incompleteness), the differences between these two results are unimportant. Then the linear-Q model is formally preferable because of its extremely low computer-time requirements. Additional reasons for the linear Q, viz. the observational evidence, is beyond the scope of this note. It is also to mention that the linear-Q model with a given value of q is equivalent to the constant-Q model and a narrow spectral band centered at frequency $f_r = Q/q$.

Although we concentrated on the constant-Q and the linear-Q models, the other power-law Q models can be included in equation (4) easily, thus yielding approximate corrections analogous to (7a) and (7b).

The cost we pay for simplicity of the present method is the restriction to SH waves and spatially constant Q. The latter assumption seems to be most critical if the Q value typical for sediments is assumed also for the underlying rocks. However, even in situations like that, the present method can give quite good results in

practical applications mentioned above, provided the excitation wavefront is placed close to the sedimentary structure at the initial time. Such an arrangement, together with a relatively large rock velocities, makes the travel time beneath the sediments very small. Therefore, the influence of the improperly modeled Q in the rock is small.

Conclusions

The practical significance of the approximate absorption correction, suggested in this paper, is threefold. (a) The correction needs a minimum programming effort and it is very fast. (b) The correction is independent of the method used for computing the perfectly elastic response. In particular, any method providing the complete wave field, not separated into elementary waves, can be used. (c) Once the perfectly elastic response has been computed, e.g., by the finite-difference method, it can be repeatedly corrected with several absorption models or several Q values. A considerable saving of computer time can be achieved in this way as compared to repeated computations including the absorption directly. Such repeated absorption corrections are very often needed in practice, since absorption parameters of geological structures are rarely known well.

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Appendix

The integral in equation (7) can be written as

$$P(f) = \int (Reh(t', f) + i\ Im\ h(t', f))\exp\{-2\pi ift'\}\ dt'. \qquad \text{(A1)}$$

It cannot be computed in a single run of the fast Fourier transform (FFT), since the function appearing in front of the exponential depends on f. At a fixed frequency,

say $\bar{f}$, we must compute two integrals of the type

$$I(\bar{f}) = \int k(t', \bar{f})\exp\{-2\pi i\bar{f}t'\}\, dt', \tag{A2}$$

where k stands for Re h, or Im h. The calculation should be done for $\bar{f} = 0$, $\Delta f, \ldots$ etc., up to a maxium significant frequency, and padded by zeros up to the Nyquist frequency. An efficient way of doing that is the following recursive formula (Dobeš, 1982).

Consider discrete values of $x_1 = k(0, \bar{f})$, $x_2 = k(\Delta t, \bar{f}), \ldots, x_n = k((n-1)\Delta t, \bar{f})$ at equal time intervals Δt, those used in computing the nonabsorbing response. If now we introduce the coefficients $b_{-1} = b_0 = 0$, and

$$b_i = b_{i-1}2\cos(2\pi\bar{f}\Delta t) - b_{i-2} + x_{n+1-i}, \quad i = 1, 2, \ldots, n, \tag{A3}$$

then

$$\begin{aligned} Re\ I(\bar{f}) &= b_n - b_{n-1}\cos(2\pi\bar{f}\Delta t). \\ Im\ I(\bar{f}) &= -b_{n-1}\sin(2\pi\bar{f}\Delta t), \end{aligned} \tag{A4}$$

This method of computing $P(f)$ was numerically tested in Zahradník *et al.* (1990). We have compared this approach to that one in which $I(\bar{f})$ is computed by repeated FFT's (one FFT for each $\bar{f}$), and a single spectral value (at $\bar{f}$) is taken from each run. The computer times of the recursive formula (A4) were found roughly 15 times shorter than the repeated FFT's.

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